# Informalizing Formal Logic*

Antonis Kakas

Department of Computer Science, University of Cyprus, Cyprus
antonis@ucy.ac.cy

**Abstract.** This paper discusses how the basic notions of formal logic can be expressed in terms of argumentation and how formal classical (or deductive) reasoning can be captured as a dialectic argumentation process. Classical propositional logical entailment of a formula is understood via the wining arguments between those supporting the formula and arguments supporting its contradictory or negated formula. Hence both informal and formal logic are captured uniformly in terms of an argumentation and its dialectic process.

## 1 Introduction

Informal Logic is usually equated with argumentation as used in real-life everyday situations. On the other hand, formal logic is concerned with the strict and precise reasoning in mathematics and science. There are several works aiming to capture informal logic in a precise formal setting such as that found in the article "Formalizing informal logic" [10] where informal logic is placed in the formal argumentation framework setting of the Carneades Argumentation System [3].

This paper is concerned with the other direction of linking formal logic to informal logic - taking informal logic as synonymous to argumentation. The aim is to reconstruct formal logic entirely in terms of argumentation enabling us to view formal deductive reasoning of classical logic as a process of dialectic argumentation.

The paper rests on the technical work of Argumentation Logic [5, 6] where this reformulation of classical Propositional Logic in terms of argumentation is carried out in formally precise terms. This work is based on notions coming from the fairly recent development of argumentation theory in Artificial Intelligence. The purpose of this paper is to unravel the technical results and present them in a generally accessible way thus providing a uniform argumentation view of both informal and formal logic.

Informalizing formal logic will be possible as a limiting case of "strict dialectic argumentation" where the arena of arguments together with the notions of counter-argument and defending argument are tightly fixed. This rigidity of the argumentation framework is to be expected since our task is to recover strict formal reasoning. The importance though of this reformulation of formal logical reasoning is that the strictness in the argumentation framework can be

* A full version of this paper is in preparation.

subsequently relaxed in cases where this is appropriate, as for example in commonsense reasoning. As a result we have a uniform way of capturing both formal and informal reasoning, smoothly moving from one to the other.

## 2 Logical Arguments

The construction of arguments in informal logic typically follows some accepted **argument schemas** that would link premises to a conclusion or a position of the argument. **Logical arguments** are arguments whose link between premises and supporting position rests on a precise logical proof in some formal logical system such as Classical Logic[1]. Hence to informalize formal logic one starts by considering the set of proof rules in a logical proof system as argument schemes. **Arguments** can then be identified with sets of logical formulae that under some of the proof rule argument schemes derive and thus support a conclusion or position of the argument. The chosen proof rule argument schemes are called **direct argument schemes**. The support of a conclusion $\phi$ by an argument $A$ is given through a **direct derivation** of $\phi$ from $A$. This will be denoted by $A \vdash_{DD} \phi$ where $DD$ denotes the chosen set of proof rules.

There are two important conditions that need to be applied to this choice of proof rule argument schemes. The first is that these argument schemes of proof rules need to be considered as **strict schemes**, i.e. that arguments constructed under these can not be defeated by questioning the validity of the chosen proof rules. The other condition has a more technical nature and requires that the proof rules of **Reduction ad Absurdum (RA)** are excluded from this initial choice of core argument schemes. This rests on the observation that the RA rules contain an element of evaluation of arguments, as they rest on first recognizing that their posited hypothesis (or argument) is inconsistent (invalid), and hence cannot be considered as a primary scheme of construction of arguments.

The main technical task then in the re-formulation of formal logic in argumentation terms is to recover at the semantic level of argumentation the RA proofs of formal logic.

Let us illustrate these ideas with a simple example. Suppose that the premises of propositional logic theory, $T$, are given by:

$$q \rightarrow \neg p \quad (1)$$

$$r \rightarrow \neg p \quad (2)$$

Given additional premises about $q$ and $r$ we can construct arguments for and against $p$. For example, if in addition we are given $q$ in $T$ then we can construct an argument $\mathbf{A}_1$ with premises the sentences (1) and $q$ supporting the conclusion

[1] We will confine ourselves to the case of Classical Logic and more specifically to classical Propositional Logic although the ideas presented would apply more generally to other formal logics.

$\neg p$, as there is a direct derivation (using the proof rule of Modus Ponens) of this conclusion from these premises.

Given this theory $T$ of premises to construct arguments that support $p$ we would need to base these on formulae that are outside $T$. We will call such premises **hypotheses** and arguments that are build on them **hypothetical arguments**. For example, we can simply build an argument $\mathbf{A}_2$ supporting $p$ based on the hypothesis of itself. We will see below the significance of this difference in the type of premises used when we consider the argumentation process between arguments. As expected, arguments whose premises are entirely drawn from the given theory will be stronger or preferred to hypothetical arguments allowing for example $\mathbf{A}_1$ to win over $\mathbf{A}_2$ and as a result the theory $T$ to logically conclude $\neg p$.

## 3 Logical Reasoning as Dialectic Argumentation

In an argumentation framework, given a position of interest we can distinguish **pro arguments** and **con arguments**, i.e. arguments that support the position and arguments that oppose the position. Arguments from these different classes **attack** each other or are **counter-arguments** of each other based on some form of conflict between them[2]. In formal logic contradiction is capture via the conflict between formulae and their negation. Generally, this (symmetric) attack relation for formal propositional logic can be captured through the joint direct derivation of an inconsistency, namely of any formula and its negation, normally denoted by $\perp$. So two arguments $\mathbf{A}_1$ and $\mathbf{A}_2$ **attack each other** if and only if $A_1 \cup A_2 \vdash_{DD} \perp$.

We will then view formal logical reasoning as a dialectic argumentation process for and against formulae and their negation. Arguments will be evaluated with respect to the other arguments that can be constructed and in particular evaluated against their counter-arguments. Arguments are **acceptable** when they exhibit a good dialectic quality, namely that they **can defend against all attacking arguments**. Analogously, an argument is **non-acceptable** if there is at least one **counter-argument that it cannot be defend against**.

To turn this into a precise definition that would then capture the strict logical reasoning of propositional logic we notice that the defence argument against any counter-argument must also be required to be acceptable and importantly to be acceptable within the context of the original argument that we want to be acceptable. Thus the central notion of **acceptability** of arguments is a relative notion that is recursively specified by (here $S$ and $S_0$ are sets of arguments):

[2] In Artificial Intelligence the attacking relation in an argumentation framework [4, 2] often contains more information than simply this symmetric incompatibility of the arguments involved. This extra information, as we will see below, pertains to the relative strength or preference of the arguments involved in the attack.

"$S$ is acceptable w.r.t. $S_0$ if for any attacking argument, $\mathbf{A}$ against $S$ there exits a defending argument $D$ that is acceptable w.r.t. $S_0$ extended by $S$."

Analogously, for the non-acceptability of $S$ w.r.t. $S_0$ we need to have an attacking argument whose all possible defences are recursively non-acceptable w.r.t. $S_0$ extended by $S$.

The **defence relation** between arguments normally captures the relative strength or preference between arguments. An argument can defend against a counter-argument if it is preferred over the attacking argument or they are non-comparable in preference. The preference and its defence relation in many domains of argumentation comes from domain specific information. Nevertheless for the quite general framework of logical reasoning as captured by Argumentation Logic [5] the preference and ensuing defence relation is minimal. It consists of two elements:

- Arguments which are entirely made out of premises in the given theory $T$ are **strictly preferred** over arguments that contain hypothetical sentences and thus can be defended against only by other arguments that also consist entirely from premises in $T$.
- A hypothetical formula $\phi$ and its complement $\phi_c$ are **equally preferred**. We are free to choose equally between the two (provided that one is not also a direct consequence of the given theory $T$) with this choice allowing us to take the side appropriately needed to defend against attacks.

Note that when the given premises $T$ are consistent the first element of defence means that attacking arguments that are made entirely from $T$ can not be defended against. Hence an argument that is attacked by an argument made entirely of premises in $T$ cannot be acceptable. Similarly, an argument $S$ made entirely from $T$ is attacked only by arguments containing hypothetical formulae and so can always be defended against by $S$. Hence such arguments are always acceptable.

In the simple example given above, we can then see that $p$ is acceptably supported, given that the premises $T$ contains the sentences (1), (2) and $q$.

To illustrate a more complex case of the dialectic argumentation process and how this captures formal logical conclusions of PL, let us consider that instead of $q$ we have the premise:

$$\neg q \rightarrow r \tag{3}$$

Hence we are now considering the theory $T$ consisting of sentences (1), (2), and (3). The position of $p$ can only be supported by arguments that contain this as a hypothesis or directly derive this from a set of formulae that contains hypotheses. Then the non-acceptability of such arguments can be determined by considering the counter-argument consisting of the premises 1 from $T$ and hypothesis $\{q\}$. The dialectical process of argumentation that shows that this attack cannot be defended against is depicted in figure 1 where for simplicity we only show the hypothesis part of the arguments involved[3].

$\{p\}$
$\uparrow$ attack : $\mathcal{T}\cup\{p\}\cup\{q\}\vdash_{DD}\bot$
$\{q\}$
$\Uparrow$ defence by opposing the hypothesis
$\{\neg q\}$
$\uparrow$ attack : $\mathcal{T}\cup\{\neg q\}\cup\{p\}\vdash_{DD}\bot$
$\{p\}$

**Fig. 1.** Dialectic process of argumentation for determining the non-acceptability of $p$, with respect to the empty set of arguments, given $T = \{(1),(2),(3)\}$, in order to determine the classical entailment of $\neg p$ from $T$. Arguments are shown only by their hypotheses as indicated in brackets.

The informal reading of this figure is as follows: The argument of supposing $p$ is attacked by the hypothesis $q$. The canonical objection or defence to this counter-argument is to assume $\neg q$. But this defence is in conflict with the argument $p$ that it is meant to be defending as together they directly derive through (2) and (3) an inconsistency. Hence although in general there is a defence against the objection (by taking the opposite view) this is not possible in the context of the particular argument that we wish to be acceptable.

This example illustrates how defences against attacks to an argument must "hold well together" in the sense that they need to be conflict free or in other words they should not contain an internal attack or counter-argument relation between them. None of the defences should form a con argument against any of the other defences and of course against the original argument of interest. In general, for any acceptable argument there must exist a set of defences against all its attacks that is attack free, i.e. directly consistent under $\vdash_{DD}$.

This rationality property of the set of defences points to the connection between acceptability of arguments and the formal logical notion of **satisfiability** of the formulae composing the argument. In fact, in the case where the given premises $T$ are classically consistent and we have a model for the set of formulae in an argument $A$ then we can choose the defences for $A$ from the set of formulae that are made true in this model and hence $A$ would be acceptable. In other words, satisfiability implies acceptability and vice versa and thus for classically consistent premises $T$ formal logical entailment coincides with **sceptical argumentation** reasoning given by: a formula $\phi$ is sceptically concluded by argumentation if and only if $\phi$ is dialectically acceptable (w.r.t. the empty set of initially accepted formulae) and $\neg\phi$ is dialectically non-acceptable. This

[3] For simplicity we also assume here that $\vdash_{DD}$ contains only the Modus Ponens proof rule argument schema.

then gives the logical equivalence of formal classical (propositional) logic and argumentation logic thus informalizing formal logic.

## 4 Beyond Classical Reasoning

Propositional Logic or full Classical Predicate Logic are not equipped or designed to deal with contradictory information. When the given premises $T$ are inconsistent formal classical reasoning collapses where every formula is trivially entailed. In contrast, argumentation is concerned exactly with how to deal with conflicting information and positions. Hence the argumentation based reformulation of formal classical reasoning that we have described above, mainly for the case of consistent premises $T$, would or should carry through when $T$ becomes classically inconsistent.

Consider in our example that we are given $p$ as an additional premise to those of $\{(1), (2), (3)\}$ that we already have. This turns the set of premises classically inconsistent. But the argumentation-based formulation of logic that we have described above will not trivialize. For example, it would sceptically conclude that $p$ holds, without also concluding that $\neg p$ holds as PL does.

The dialectic argumentation proof in figure 1, that gave us earlier the conclusion that $p$ is non-acceptable (and hence that $\neg p$ holds since also it is acceptable), now changes. Indeed, we now have another way to defend against the attack(s) containing the hypothesis $q$. We can now defend using the premise $p$ that, as we have explained above, is preferred over arguments that contain hypotheses as it is made purely of sentences in the given premises $T$. On the other hand, any argument supporting $\neg p$ will be attacked by the argument made purely from the premise $p$. But this cannot be defended against since there is no direct or explicit information to its contrary in the premises. Hence $p$ is acceptable and any argument supporting $\neg p$ is not acceptable, thus $p$ is sceptically concluded. Another example of escaping from the trivialization of formal classical reasoning would be the case where we have in the premises $T$ both $p$ and $\neg p$. Then each of these premises would defend against each other and so both arguments would be acceptable and therefore there will be no sceptical conclusion for whether $p$ holds or not.

The strict conditions on the argumentation framework that we have imposed so that we can match the strict reasoning of formal logic can be further relaxed by allowing the premises themselves to take a defeasible nature, e.g. implications to represent only a “normally or mostly” nature of associating their condition with their conclusion. Then relative preferences amongst this defeasible knowledge enriches the defence relation by rendering some arguments stronger and hence able to defend against (some of) their counter-arguments but not vice-versa.

This is particularly appropriate when we consider the informal logic of **common sense human reasoning** [9, 8, 1] where people normally reason within a context and where the common sense knowledge is in this form of loose associations. General or individual **human biases** give preference to some of these statements and we can then understand common sense reasoning in argumentation terms in the same way we have expressed formal logical reasoning. Argumentation thus gives a uniform umbrella framework covering the whole spectrum of reasoning from the very strict formal reasoning to the flexible informal reasoning.

## 5 Conclusions

We have described how formal classical reasoning can be captured through the same process of dialectic argumentation that is normally associated with informal logic. This reformulation of logic in terms of argumentation has been shown [6] to be complete for propositional logic. The same approach can be applied more generally to first order predicate logic. An interesting example of this is that of Aristotelian syllogisms when these are seen as canonical forms of strict classical reasoning of predicate logic. Furthermore, syllogisms have been studied as an example of human cognitive reasoning, see e.g. [7], where it is observed that humans do not reason according to formal logic but that their interpretation of syllogism is indeed a case of informal logic. In a recent challenge[4] to model the cognitive syllogistic reasoning of humans, argumentation was shown as a promising approach towards this goal.

By varying the degree of flexibility within the argumentation framework and its dialectic process we can move from formal logic to informal logic and back. Argumentation thus provides a way to unify these two worlds of logic, normally considered as very different, under the same conceptual framework. It provides a uniform umbrella framework covering the whole spectrum of reasoning from the very strict formal reasoning to the extremely flexible informal human reasoning.


## Acknowledgement

I would like to thank Loizos Michael and Francesca Toni for their continued collaboration on argumentation. This has been very useful in writing this paper.


## References


1. I.-A. Diakidoy, A., L. Michael, and R. Miller. Story Comprehension through Argumentation. In *Proceedings of the 5th International Conference on Computational Models of Argument (COMMA)*, pages 31–42, 2014.
2. P. M. Dung. On the Acceptability of Arguments and its Fundamental Role in Nonmonotonic Reasoning, Logic Programming and n-person Games. *Artificial Intelligence*, 77:321–357, 1995.
3. Thomas F. Gordon and Douglas Walton. The carneades argumentation framework - using presumptions and exceptions to model critical questions. In *Computational Models of Argument: Proceedings of COMMA 2006, September 11-12, 2006, Liverpool, UK*, pages 195–207, 2006.
4. A. Kakas, R. Kowalski, and F. Toni. Abductive Logic Programming. *Journal of Logic and Computation*, 2(6):719–770, 1992.


[4] This challenge was announced at https://www.cc.uni-freiburg.de/syllogchallenge.

5. A. Kakas, F. Toni, and P. Mancarella. Argumentation Logic. In *Proceedings of the 5th International Conference on Computational Models of Argument (COMMA)*, pages 12–27, 2014.
6. Antonis C. Kakas, Paolo Mancarella, and Francesca Toni. On argumentation logic and propositional logic. *Studia Logica*, Jul 2017.
7. Sangeet Khemlani and P. N. Johnson-Laird. Theories of the syllogism: A meta-analysis. *Psychological Bulletin*, 138:427–457, 2012.
8. R. Kowalski. *Computational Logic and Human Thinking: How to Be Artificially Intelligent*. Cambridge University Press, New York, NY, USA, 2011.
9. K. Stenning and M. van Lambalgen. *Human Reasoning and Cognitive Science*. MIT Press, 2008.
10. Douglas Walton and Thomas F. Gordon. Formalizing Informal Logic. *Informal Logic*, 35(4):508–538, 2015.