УДК 517.937, 517.928.2, 519.217.2

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ИСПОЛЬЗОВАНИЕ ПРИБЛИЖЕНИЯ СРЕДНЕГО ПОЛЯ ДЛЯ АНАЛИЗА КРУПНОМАСШТАБНЫХ ТРАНСПОРТНЫХ СЕТЕЙ С МАЛЫМ ПАРАМЕТРОМ*

Аннотация

Решение задач математического моделирования сложных транспортных сетей на данном этапе представляет большую сложность по причине большого объема данных, которые приходится анализировать. Например, огромное количество возможных вариантов перевозок затрудняет получение достаточно экономного плана эмпирическим или экспертным путем. Применение математических методов и использование современных вычислительных алгоритмов в планировании перевозок дает большой экономический эффект. Проведенный анализ показал, что этот подход является эффективным для решения широкого круга технических и технологических проблем проектирования, строительства и функционирования транспортных систем. В рамках этого подхода удается создать эффективный алгоритм минимизации затрат на проектирование, строительство и эксплуатацию таких систем. Транспортные задачи могут быть решены симплексным методом, однако матрица системы ограничений транспортной задачи часто настолько сложна, что для ее решения разработаны специальные методы. В данной работе исследуются крупномасштабные транспортные сети с использованием приближения среднего поля Добрушина. Показано, что анализ эволюции крупномасштабных транспортных систем можно описать с помощью системы дифференциальных уравнений бесконечного порядка. Для этой системы можно поставить задачу Коши тихоновского типа с малым параметром, который вносит сингулярное возмущение. В статье доказана теорема существования решения этой задачи Коши.

Ключевые слова

Аналитические методы в теории транспортных сетей; системы дифференциальных уравнений бесконечного порядка; малый параметр; счетные цепи Маркова; крупномасштабные транспортные сети; приближение среднего поля Добрушина; транспортная задача; динамика сложных систем.

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MEAN-FIELD APPROXIMATION FOR LARGE-SCALE TRANSPORT NETWORKS WITH A SMALL PARAMETER

Abstract

The solution of mathematical simulation problems of complex transport networks at this stage is more difficult because of the large amount of data that must be analyzed. For example, a huge number of possible options of traffic makes it difficult to obtain sufficient economical plan through empirical or using expert approach. Application of mathematical methods and use of modern computational algorithms for transport planning gives considerable economic benefit. It is shown this approach is effective for solving a wide range of technical and technological problems of design, construction and operation of transport systems. In this approach manages to create an efficient algorithm for minimizing the cost of design, construction and operation of such systems. The transportation problem can be solved by simplex method but the matrix of the constraints of the transportation problem is often so complex that its solution developed special methods. In this paper it is studied

* **Труды II Международной научной конференции «Конвергентные когнитивно-информационные технологии» (Convergent'2017), Москва, 24-26 ноября 2017**

Proceedings of the II International scientific conference "Convergent cognitive information technologies" (Convergent'2017), Moscow, Russia, November 24-26, 2017

large-scale transport network using Dobrushin's mean-field approximation. It is shown that the analysis of the evolution of large-scale transport systems can be described using systems of differential equations of infinite order. For this system, it is formulated the Cauchy problem Tikhon type with a small parameter ε, which introduces a singular perturbation. The theorem of existence of the solution of this Cauchy problem is proved.

Keywords

Analytical methods in transport networks theory; systems of differential equations of infinite order; small parameter; countable Markov chains; large-scale transport networks; Dobrushin mean-field approximation; transportation problem; dynamics of complicated systems.

Introduction

In this paper large-scale transport networks are studied using Dobrushin mean-field approach [1,4-5,10,17-19]. We assume that the transport networks deal with the problem of proving the global convergence of the solutions of certain infinite systems of ordinary differential equations to a time-independent solution. In work, [4,5,10] the infinite systems of differential equations modelling large-scale transport systems are studied and the sufficient conditions of global stability and global asymptotic stability are obtained.

Cauchy problems for the systems of ordinary differential equations of infinite order was investigated A.N. Tihonov [14], K.P. Persidsky [11], O.A. Zhautykov [20-21], Ju. Korobeinik [6], M.A. Krasnoselsky, P.P. Zabreyko [8], A.M. Samoilenko, Yu.V. Teplinskii [12] other researchers.

It was studied the singular perturbed systems of ordinary differential equations by A.N. Tihonov [15], A.B. Vasil'eva [16], S.A. Lomov [9] other researchers.

In papers [3], [7], [13] the authors built various models of large-scale queueing systems and considered their dynamics.

In paper [2] it was investigated the singular perturbed systems of ordinary differential equations of infinite order of Tikhonov-type $\varepsilon\dot{x} = F(x(t,g_x), y(t,g_y), t)$, $\dot{y} = f(x(t,g_x), y(t,g_y), t)$ with the initial conditions $x(t_0) = g_x$, $y(t_0) = g_y$, where $x, g_x \in X$, $X \subset l_1$ and $y, g_y \in Y$, $Y \in R^n$, $t \in [t_0, t_1]$ ($t_0 < t_1$), $t_0, t_1 \in T$, $T \in R$, g_x and g_y are given vectors, $\varepsilon > 0$ is a small real parameter.

In this paper we considered large-scale transport network systems that consists of infinite number of network service nodes with a Poisson input flow of requests. We assume that the queuing system has N nodes and rN servers. At each node (N nodes) the arrivals of particles form a Poisson flow of rate λ. For an empty node a particle leaves the system. A server at the node takes the particle and moves to a random node. Travelling time is exponential of mean $1/\mu$. The number of servers at each node (of these N nodes) is bounded by m. We consider the property of the system for the limiting deterministic process as $N \to \infty$. The evolution analysis of large-scale transport systems can be described using an infinite system of differential equations. It is possible to investigate Tikhonov type Cauchy problem for this system with small parameter. In this paper we apply Dobrushin mean-field approaches from [5,10] for analysis of the singular perturbed systems of ordinary differential equations of infinite order.

It is possible to formulate Tikhonov type Cauchy problem for this system with small parameter ε and initial conditions. We study the singular perturbed Tikhonov systems of ordinary differential equations of infinite order $\dot{u} = f(u(t,g_u), U(t,g_U), t)$, $\varepsilon\dot{U} = F(u(t,g_u), U(t,g_U), t)$ with the initial conditions $u(0,g_u) = g_u$, $U(0,g_U) = g_U$, where $\varepsilon > 0$ is a small real parameter. The theorem of existence of solution for this Cauchy problem is proved.

Large-scale transport network model

Let's consider a large-scale transport networks that consist of N nodes, a virtual node and rN servers. At each node (of these N nodes) the arrivals of particles form a Poisson flow of rate λ. If a particle arrives at an empty node then the particle leaves the system. Otherwise, if there is a server at the node then the server takes the particle and jumps to the virtual node. At this node the server waits for an exponential time of mean $\bar{t} = 1/\mu$. After the server jumps to a random node with uniform distribution. If the number of servers at the chosen node equals m then the server waits for the following attempt at the virtual node. The non-negative number of servers at each node (except virtual) is bounded by m. Consider the fractions $f_k = n_k / N$, $V = W / N$, where n_k is the (random) number of nodes with k servers and W is the number of servers at the virtual node. It is more convenient to regard the tail probabilities $u_k = \sum_{i=k}^{m} f_i$. The state space of the corresponding Markov process $U_N(t) = (u_k(t), V(t))$ is the set X_N of all vectors $u = (u_1, \ldots, u_m, V)^T$ in $(1/N)Z_+^{m+1}$ such that $1 = u_0 \ge u_1 \ge \ldots \ge u_m$,

$V \geq 0,\ u_1 + \ldots + u_m + V = r$. The generator of $U_N(t)$ is the operator $A_N(t)$ acting on functions and given by

$$A_N(t) f(u) = N\lambda \sum_{k=1}^{m-1} (u_k - u_{k+1})[f\left(u - \frac{e_k}{N} + \frac{e_{m+1}}{N}\right) - f(u)] +$$
$$+ N\lambda u_m [f\left(u - \frac{e_k}{N} + \frac{e_{m+1}}{N}\right) - f(u)] +,$$
$$+ N\mu V \sum_{k=1}^{m} (u_{k-1} - u_k)[f\left(u - \frac{e_k}{N} + \frac{e_{m+1}}{N}\right) - f(u)]$$

where e_k denotes a vector with the component of number k equal to 1 and others equal to 0.

The mean-field approximation suggests that the whole process $U_N(t)$ is asymptotically deterministic as $N \to \infty$. More precisely, let X denote the set of all R^{m+1} vectors defined by (1). Then, if the distribution of the initial state $U_N(0)$ converges to the Dirac delta-measure concentrated at some point $g \in X$, the distribution of $U_N(t)$ is concentrated on the orbit $u(t) \in X$ as $N \to \infty$ where $u(t)$ is the solution of the following system of differential equations (mean-field equations)

$$\begin{cases} \dot{V}(t) = \lambda u_1(t) - V(t)\ u_0(t),\ u_0(t) = 1; \\ \dot{u}_k(t) = \lambda\left(u_{i+1}(t) - u_i(t)\right) + \mu V(t)\left(u_{i-1}(t) - u_i(t)\right), \\ \sum_{k=0}^{\infty} u_k(t) + V(t) = r(t),\ r(r) > 0, \\ V(0) = V_0 \geq 0,\ u_k(0) = g_k \geq 0,\ k = 0,1,2,\ldots, \\ 1 = g_0 \geq g_1 \geq g_2, \ldots,\ t \geq 0 \end{cases}$$

where $r(t) > 0$ is a parameter and $g = \{g_k\}_{k=1}^{\infty}$ is a numerical sequence. The infinite order system (2) is non-linear and its right-hand side depends on time.

Large-scale queueing systems model with a small parameter

We can investigate infinite system of differential equations with small parameter such form

$$\begin{cases} \dot{V}(t) = \lambda u_1(t) - V(t)\ u_0(t),\ u_0(t) = 1; \\ \dot{u}_k(t) = \lambda\left(u_{k+1}(t) - u_k(t)\right) + \mu V(t)\left(u_{k-1}(t) - u_k(t)\right),\ k = 1,2,\ldots,n, \\ \varepsilon^{s_k} \dot{u}_k(t) = \lambda\left(u_{k+1}(t) - u_k(t)\right) + \mu V(t)\left(u_{k-1}(t) - u_k(t)\right),\ k = n+1, n+2, \ldots, \\ \sum_{k=0}^{\infty} u_k(t) + V(t) = r(t),\ r(t) > 0, \\ V(0) = V_0 \geq 0,\ u_k(0) = g_k \geq 0,\ k = 0,1,2,\ldots, \\ 1 = g_0 \geq g_1 \geq g_2, \ldots,\ t \geq 0, \end{cases}$$

where ε is a small parameter that bring a singular perturbation to the system (2) which allows us to describe the processes of rapid change of the systems and $s = \{s_k\}_{k=n+1}^{\infty} (s_k > 0)$ is a numerical sequence.

Using (3) we can write Tikhonov problems for systems of ordinary differential equations of infinite order with a small parameter ε and initial conditions

$$\begin{cases} \dot{V}(t) = \lambda u_1(t) - V(t)\ u_0(t),\ u_0(t) = 1; \\ \dot{u} = f(u(t,\mu,\lambda,g_u), U(t,\mu,\lambda,g_U), t), \\ \varepsilon^{s_k} \dot{U} = F(U(t,\mu,\lambda,g_U), t); \\ V(0) = V_0 \geq 0,\ u(0,\mu,\lambda,g_u) = g_u, \\ U(0,\mu,\lambda,g_U) = g_U, \end{cases}$$

where $u, f \in X$, $X \in R^{n+1}$ are *(n+1)*-dimensional functions; $U, F \in Y$, $Y \subset l_1$ are infinite-dimensional functions and $t \in [0, T_0]$ ($0 < T_0 \leq \infty$), $t \in T$, $T \in R$; $g_u \in X$ and $g_U \in Y$ are given vectors

$(g_u = \{g_k\}_{k=0}^{n}, g_U = \{g_k\}_{k=n+1}^{\infty}, 1 = g_0 \geq g_1 \geq g_2, \ldots)$, $\varepsilon > 0$ is a small real parameter; $u(0, g_u) = g_u$ and $U(0, g_U) = g_U$ are the conditions for solutions of (4). Given functions $f(u(t,\mu,\lambda,g_u),U(t,\mu,\lambda,g_U),t)$ and $F(U(t,\mu,\lambda,g_U),t)$ are continuous functions for all variables

$$f_k(u(t,\mu,\lambda,g_u),t) = \lambda\left(u_{k+1}(t) - u_k(t)\right) + \mu V(t)\left(u_{k-1}(t) - u_k(t)\right), k = 1,\ldots,n,$$

$$F_k(U(t,\mu,\lambda,g_U),t) = \lambda\left(U_{k+1}(t) - U_k(t)\right) + \mu V(t)\left(U_{k-1}(t) - U_k(t)\right), k = n+1, n+2,\ldots$$

Let S is an integral manifold of the system (4) in $X \times Y \times T$. If any point $t^* \in [0, T_0]$ $(u(t^*), U(t^*), t^*) \in S$ of trajectory of this system has at least one common point on S this trajectory $(u(t,G),U(t,g),t) \in S$ belongs the integral manifold S totally.

If we assume in (4) that $\varepsilon = 0$ than we have a degenerate system of the ordinary differential equations and a problem of singular perturbations

$$\begin{cases} \dot{V}(t) = \lambda u_1(t) - V(t)\ u_0(t),\ u_0(t) = 1, \\ \dot{u} = f(u(t,\mu,\lambda,g_u),U(t),t), \\ \quad 0 = F(u(t,\lambda,g_u),U(t,\mu,\lambda),t); \\ \qquad u(0,\lambda,g_u) = g_u, \end{cases}$$

where the dimension of this system is less than the dimension of the system (4), since the relations $F(u(t,\lambda),U(t,\lambda),\lambda,t) = 0$ in the system (6) are the algebraic equations (not differential equations). Thus for the system (9) we can use limited number of the initial conditions then for system (4). Most natural for this case we can use the initial conditions $u(0,\lambda,g_u) = g_u$ for the system (6) and the initial conditions $U(0,\lambda,U_y) = g_U$ disregard otherwise we get the overdefined system. We can solve the system (6) if the equation $F(u(t,\lambda),U(t,\lambda),\lambda,t) = 0$ has roots. If it is possible to solve we can find a finite set or countable set of the roots $U_q(t,\lambda,g_u) = u_q(u(t,\lambda,g_u),t)$ where $q \in N$. If the implicit function $F(u(t,\lambda),U(t,\lambda),\lambda,t) = 0$ has not simple structure we must investigate the question about the choice of roots. Hence we can use the roots $U_q(t,\lambda,g_u) = u_q(u(t,\lambda,g_u),t)$ ($q \in N$) in (10) and solve the degenerate system

$$\begin{cases} \dot{u}_d = f(u_d(t,\lambda,g_u),u_q(u_d(t,\lambda,g_u),t),\lambda,t); \\ \qquad U_d(0,\lambda,g_u) = g_u. \end{cases}$$

Since it is not assumed that the roots $U_q(t,\lambda,g_u) = u_q(u(t,\lambda,g_u),\lambda,t)$ satisfy the initial conditions of the Cauchy problem (4) ($U_q(0) \neq g_u,\ q \in N$), the solutions $U(t,\lambda,g_U)$ (4) and $U_q(t,\lambda,g_u)$ do not close to each other at the initial moments of time $t > 0$. Also there is a very interesting question about behaviors of the solutions $u(t,\lambda,g_u)$ of the singular perturbed problem (4) and the solutions $u_d(t,\lambda,g_u)$ of the degenerate problem (6). When $t = 0$ we have $u(0,\lambda,g_u) = u_d(0,\lambda,g_u)$. Do these solutions close to each other when $t \in (0, T_0]$? The answer to this question depends on using roots $U_q(t,\lambda,g_u) = u_q(u(t,\lambda,g_u),t)$ and the initial conditions, which we apply for the systems (7).

Analysis of infinite order system of differential equations

We can rewrite Tikhonov problems (4) for systems of ordinary differential equations of infinite order with a small parameter ε and initial conditions in the form

$$\begin{cases} \dot{v} = F_R(v(t,\mu,\lambda,\varepsilon,v^0),t), \\ \quad v(0,\mu,\lambda,\varepsilon,v_0) = v^0, \end{cases}$$

where

$$v = (V, u_0, u_1, \ldots, u_n, U_{n+1}, U_{n+2}, \ldots),$$

$$F_{R0} = \lambda u_1(t) - V(t)\ u_0(t),$$

$$F_{Rk} = \lambda\left(u_{k+1}(t) - u_k(t)\right) + \mu V(t)\left(u_{k-1}(t) - u_k(t)\right), k = 1,\ldots,n,$$

$$F_{Rk} = \varepsilon^{-s_k}\lambda\left(u_{k+1}(t) - u_k(t)\right) + \varepsilon^{-s_k}\mu V(t)\left(u_{k-1}(t) - u_k(t)\right), k = n+1, n+2,\ldots,$$

$$v^0 = (V_0, g_u, g_U),$$

where $v_0^0 = V,\ v_k^0 = g_k, k = 1,2,\ldots$

Using methods from [12], [20-21] we can consider Tikhonov-type problems (8)

$$\begin{cases} \dot{v} = F_R(v_0, v_1, \dots, v_n, \dots, \mu, \lambda, \varepsilon, t), \\ v(0, \mu, \lambda, \varepsilon, v_0) = v_0, \end{cases}$$

Definition. A function $F_R\left(v_0, v_1, \dots, v_n, \dots, \mu, \lambda, \varepsilon, t\right)$ is called strongly continuous if for any $\varepsilon_0 > 0$, there exist N_0 and $\delta_0 > 0$ such that the inequality $|v_i^{'} - v_i^{''}| < \delta_0$, $i = 0,1,2,\dots,\ N_0$, implies the estimate for any $\mu \geq 0, \lambda \geq 0, \varepsilon > 0$

$$|F_R\left(v_0^{'}, v_1^{'}, \dots \mu, \lambda, \varepsilon\right) - F_R\left(v_0^{''}, v_1^{''}, \dots, \mu, \lambda, \varepsilon\right)| < \varepsilon_0.$$

Theorem. *Assume that the right-hand sides of the system of equations (10)*

- *are defined for any* $v_i(\mu, \lambda, \varepsilon, t) \in R^1, i = 0,1,2,\dots$, $\mu \geq 0, \lambda \geq 0, \varepsilon > 0$ *and all* $t \in T_0 = [0, \Delta t] \subset R^1$;
- *are strongly continuous in* $v_0, v_1, \dots$ *for fixed* $t \in T_0$, $\mu \geq 0$, $\lambda \geq 0$, $\varepsilon > 0$ *and measurable in* $t \in T_0$ *for fixed* $v_i(\mu, \lambda, \varepsilon, t), i = 0,1,2,\dots$;
- *satisfy the inequalities*

$$|F_{Ri}\left(t, v_0, v_1, \dots, \mu, \lambda, \varepsilon\right)| < M_i(t)$$

for all $i = 0,1,2,\dots$, *where* $M_i(t)$ *are functions summable on the segment* T_0 *and for any* $\mu \geq 0, \lambda \geq 0, \varepsilon > 0$.

Then, for any vector $\left(v_0^0, v_1^0, \dots\right)$ *with real coordinates, there exists at least one solution* $\left(v_0(\mu, \lambda, \varepsilon, t), v_1(\mu, \lambda, \varepsilon, t), \dots\right)$ *of the system of equations (14) such that* $v_i(0) = v_i^0, i = 0,1,2,\dots$.

Proof. We replace the system of equations (8) by the following system of integral equations:

$$v_i(t) = v_i^0 + \int_0^t F_{Ri}\left(t, v_0(t), v_1(t), \dots, \mu, \lambda, \varepsilon\right) dt, i = 0,1,2,\dots,$$

and consider a mapping (A)

$$z_i(t) = v_i^0 + \int_0^t F_{Ri}\left(t, v_0(t), v_1(t), \dots, \mu, \lambda, \varepsilon\right) dt, i = 0,1,2,\dots,$$

which establishes a correspondence between an arbitrary countable system of continuous functions $\{v_i(t)\}_{i=0}^{\infty}$ and another system of this sort $\{z_i(t)\}_{i=0}^{\infty}$. Note that if $F_R(t, v_0, \dots, v_n, \mu, \lambda, \varepsilon)$ is a continuous function of finitely many variables $\{v_i(t)\}_{i=0}^{n}$ measurable with respect to t for fixed $v_i, i = \overline{0,n}$, then the function

$$\Phi(t) = F_R(t, \phi_0(t), \dots, \phi_n(t), \mu, \lambda, \varepsilon)$$

is measurable if $\phi_i(t), i = \overline{0,n}$, are measurable.

Thus, the function

$$\Psi_n(t) = F_R(t, \phi_0(t), \dots, \phi_n(t), 0, 0, \dots, \mu, \lambda, \varepsilon)$$

is measurable and, therefore, the function

$$F_R(t, \phi_0(t), \phi_1(t), \dots, \mu, \lambda, \varepsilon) = \Psi(t, \mu, \lambda, \varepsilon)$$

is also measurable because

$$\Psi(t) = \lim_{n \to \infty} \Psi_n(t, \mu, \lambda, \varepsilon),$$

which readily follows from the condition of strong continuity. The requirement of summability follows from condition 3 of Theorem. We consider a system of functions $\{v_i(t)\}_{i=0}^{\infty}$ as a point P of an abstract space R. If there exists a point P invariant under mapping (A) (14), then it specifies a solution of the system of equations (13) and, hence, of system (10).

Consider a set M_0 formed by three points P for which $\{v_i(t)\}_{i=0}^{\infty}$ satisfy the conditions

$$|v_k(t) - v_k^0| \leq \int_0^t M_k(t) dt, \quad |v_k(t') - v_k(t'')| \leq \int_{t'}^{t''} M_k(t) dt, k = 0,1,2,\dots.$$

It is easy to see that mapping (A) (14) maps the set M_0 into itself. We now introduce mapping (B) by putting every point P in correspondence with a set of numbers

$$\frac{a_0^0}{N_0}, \dots, \frac{a_0^n}{N_0}, \dots,$$
$$\dots\dots,$$

$$\frac{a_n^1}{nN_n},\ldots,\frac{a_n^n}{nN_n},\ldots,$$
$$\ldots\ldots,$$

where $N_i = v_i^0 + \int_0^{\Delta t} M_i(t)dt$ and the numbers $\{a_n^r\}_{n,r=0}^{\infty}$ ($a_n^0,\ldots,a_n^n,\ldots$) are the coefficients of the Fourier expansion of a function $v_n(t)$ in a certain complete orthogonal system of functions on the segment T_0. By ordering the set of numbers (16), we obtain a numerical sequence $b_0, b_1,\ldots,b_n,\ldots$. Moreover, we have

$$\sum_{k=0}^{\infty}\left(a_n^k\right)^2 = \int_0^{\Delta t}\left(v_n(t)\right)^2 dt \le \int_0^{\Delta t}\left(v_n^0 + \int_0^t M_k(t)dt\right)^2 dt \le$$
$$\le \int_0^{\Delta t} N_n^2 dt = aN_n^2,$$

whence it follows that

$$\sum_{i=0}^{\infty} b_i^2 = \sum_{0=1}^{\infty}\sum_{k=0}^{\infty}\left(\frac{a_n^k}{nN_n}\right)^2 \le a\sum_{n=0}^{\infty}\frac{1}{n^2} = \frac{a\pi^2}{6}.$$

Thus, mapping (B) maps the set M_0 into a subset M_0^* of the Hilbert space l_2. Therefore, mapping (A) induces a mapping (A^*) of the set M_0^* into itself. Further, if mapping (A^*) has a fixed point $P^* \in M_0^*$, then the corresponding point $P^* \in M_0$ determines the solution of equation (17) and, hence, (10). To use the Schauder theorem, it suffices to show that the set M_0^* is compact and convex. If $P'^* = (b_0^{'},\ldots,b_n^{'},\ldots)$ and $P''^* = (b_0^{''},\ldots,b_n^{''},\ldots)$ are points from M_0^*, then the point

$$\alpha P'^* + \beta P''^* = (\alpha b_0^{'} + \beta b_0^{''}, \alpha b_1^{'} + \beta b_1^{''},\ldots), \alpha + \beta = 1, \alpha > 0, \beta > 0,$$

belongs to M_0^* because it corresponds to the system of functions

$$\alpha v_0^{'}(t) + \beta v_0^{''}(t), \alpha v_1^{'}(t) + \beta v_1^{''}(t),\ldots.$$

specifying a point from the set M_0. Indeed,

$$\left|\alpha v_k^{'}(t) + \beta v_k^{''}(t) - v_k^0\right| = \left|\alpha(v_k^{'}(t) - v_k^0) + \beta(v_k^{''}(t) - v_k^0)\right| \le (\alpha+\beta)\int_0^t M_k(t)dt = \int_0^t M_k(t)dt,$$

i.e., condition 1 is satisfied. Similarly, the inequality

$$\left|\alpha v_k^{'}(t^{'}) + \beta v_k^{''}(t^{'}) - \alpha v_k^{'}(t^{''}) - \beta v_k^{''}(t^{''})\right| \le (\alpha+\beta)\int_0^t M_k(t)dt$$

implies condition 2. Hence, the set M_0^* is convex. In this set, we choose an arbitrary sequence of points P_i^*. This sequence corresponds to the sequence of points $P_i\left(v_0^{(i)}(t), v_1^{(i)}(t),\ldots\right)$ in the set M_0. According to conditions 1 and 2, the sequence $v_0^{(i)}(t), i = 0,1,2,\ldots$, is uniformly bounded and equicontinuous and, consequently, it contains a subsequence $v_0^{(\alpha_0)}(t), v_0^{(\alpha_1)}(t),\ldots,v_0^{(\alpha_s)}(t),\ldots$ that converges uniformly in $t \in T_0$. However, the sequence $v_1^{(\alpha_h)}(t),\ h \to \infty$, is also uniformly bounded and equicontinuous and, hence, it also contains a convergent subsequence

$$v_1^{(\beta_0)}(t), v_1^{(\beta_1)}(t),\ldots,v_1^{(\beta_s)}(t),\ldots.$$

This process can be continued infinitely.

We compose the table

$$v_0^{(\alpha_0)}(t) v_0^{(\alpha_1)}(t) v_0^{(\alpha_2)}(t)\ldots$$
$$v_1^{(\beta_0)}(t) v_1^{(\beta_1)}(t) v_1^{(\beta_2)}(t)\ldots$$
$$v_2^{(\gamma_0)}(t) v_2^{(\gamma_1)}(t) v_2^{(\gamma_2)}(t)\ldots$$
$$\ldots\ldots\ldots\ldots$$

and rewrite the set of sequences row by row

$$v_0^{(\alpha_0)}(t) v_0^{(\beta_1)}(t) v_0^{(\gamma_2)}(t)\ldots$$

$$v_1^{(\alpha_0)}(t)v_1^{(\beta_1)}(t)v_1^{(\gamma_2)}(t)\ldots$$
$$v_2^{(\alpha_0)}(t)v_2^{(\beta_1)}(t)v_2^{(\gamma_2)}(t)\ldots$$
$$\ldots\ldots\ldots\ldots$$

Each of these sequences converges as a subsequence of a convergent sequence supplemented by finitely many elements. Thus, the sequence of points

$$P_{\alpha_0}, P_{\beta_1}, P_{\gamma_2}, \ldots \subset M_0$$

converges weakly (coordinatewise) to a point $P_0 \in M_0$ (uniformly in $t \in T_0$). For the sake of convenience, we rewrite sequence (26) as

$$P_0, P_1, P_2, \ldots, P_n, \ldots$$

Let us show that the sequence of the corresponding points $P_0^*, P_1^*, P_2^*, \ldots$ from the set M_0^* converges to the point $P_0^* \in M_0^*$ in the norm of the Hilbert space l_2. Indeed, the distance between the points P'^* and P''^* from M_0^* is given by the formula

$$\rho\left(P'^*, P''^*\right) = \sqrt{\sum_{i=0}^{\infty}(b_i' - b_i'')^2} = \sqrt{\sum_{n=0}^{\infty}\frac{1}{n^2 N_n^2}\int_0^{\Delta t}(v_n' - v_n'')^2\,dt},$$

whence it follows that

$$\rho\left(P_0^*, P_k^*\right) \le \sqrt{\sum_{n=0}^{n_0}\frac{1}{n^2 N_n^2}\int_0^{\Delta t}(v_n^0 - v_n^k)^2\,dt + \Delta t\sum_{n=n_0}^{\infty}\frac{1}{n^2}}$$

is arbitrarily small for sufficiently large n_0 and k. This means that the set M_0^* is compact. Note that one can easily prove that mapping (B) is a homeomorphism, i.e., the sets M_0 and M_0^* are topologically equivalent. Theorem is proved.

Conclusions

We consider the property of the system for the limiting deterministic process as $N \to \infty$. The evolution analysis of large-scale transport systems can be described using an infinite system of differential equations. It is possible to formulate Tikhonov type Cauchy problem for this system with small parameter ε and initial conditions. Tikhonov type Cauchy problem for this system with small parameter ε is investigated. The theorems of existence of solutions for this Cauchy problem is proved with taking into account parameters $\lambda, \mu, \varepsilon$.

Acknowledments

The publication was prepared with the support of the "RUDN University Program 5-100" and partially funded by RFBF grants № 15-07-08795, № 16-07-00556.

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