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Дараселия А.В.[1], Сопин Э.С.[1,2]

[1] Российский университет дружбы народов, г. Москва, Россия
[2] Федеральный исследовательский центр «Информатика и управление» РАН, г. Москва, Россия

АНАЛИЗ ОДНОГО ПОДХОДА ДЛЯ СНИЖЕНИЯ ЭНЕРГОПОТРЕБЛЕНИЯ ОБЛАЧНОЙ СИСТЕМЫ*

Аннотация

Повышение энергоэффективности систем облачных вычислений является одной из важнейших задач провайдеров облачных услуг. Одним из наиболее очевидных решений для снижения энергопотребления является динамическое управление количеством работающих серверов в зависимости от нагрузки. Однако такой подход приводит к дополнительных временным и энергозатратам на подключение и отключение вычислительных мощностей. Исследования показывают, что эти дополнительные затраты могут существенно снижать выгоду от использования динамического управления. В связи с этим облачные провайдеры применяют механизмы, снижающие количество включений/выключений серверов. Один из таких механизмов исследуется в данной статье. Рассматривается облачная система, в которой сервер, не имеющий активных виртуальных машин, выключается не сразу, а по прошествии некоторого времени. Построена математическая модель системы облачных вычислений с учетом времени включения и выключения для анализа показателей энергопотребления. Проводится анализ условий, при которых имеет смысл с точки зрения энергопотребления не переводить сервер в режим ожидания сразу же как опустела очередь запросов. Были получены аналитические выражения для стационарного распределения и основных показателей производительности и энергопотребления.

Ключевые слова

Облачные вычисления; энергоэффективность; теория массового обслуживания.

Daraseliya A.V.[1], Sopin E.S.[1,2]

[1] Peoples' Friendship University of Russia, Moscow, Russia
[2] Federal Research Center Computer Science and Control of the Russian Academy of Sciences, Moscow, Russia

ANALYSIS OF AN APPROACH TO INCREASE ENERGY EFFICIENCY OF A CLOUD COMPUTING SYSTEM

Abstract

Enhancing energy efficiency of cloud computing systems is one of the key challenges for cloud providers. One of the most obvious ways to decrease energy consumption is dynamic control of switched on servers according to the system load. However, this approach leads to additional energy and time loss due to switching on/off of computing resources. Recent research shows that the additional energy and time loss may significantly decrease positive effect of dynamic control. In this regard, cloud providers employ various mechanisms that decrease server switching number. One of the mechanisms is considered in the paper. We consider a cloud computing system, in which a server does not switch off immediately, as it remains empty but after a certain time. We develop a mathematical model of a cloud computing system with switch on/off periods and analyze its energy efficiency metrics. We investigate how energy efficiency of a cloud system is affected by a waiting time before a server goes to standby mode.

Keywords

Cloud computing; cloud computing; energy efficiency; queuing system.

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1. Introduction

Energy efficiency of cloud system is very important issue for cloud providers. The servers can be put into standby state in order to improve the energy efficiency of a cloud system in case of light load. On the one hand, the switching to standby mode allows us to reduce power consumption, and on the other hand, it leads to extra power usage to turn on/off the server. Therefore, it is important to understand under what conditions it will be advantageous to put the server in standby state, and under what conditions it is more profitable to leave it in the operating mode.

In [1] we have considered a cloud system taking into account switch on and switch off periods of servers, and it was assumed that the server switches off immediately as it remains empty. In this paper, we consider a model, in which the server does not switch off immediately after it is empty, but waits for an exponentially distributed time. For simplicity, we consider only one server with a number of virtual machines working on it.

2. Mathematical model of a cloud system

We consider a multi server queuing system with C servers. Customers arrive according to the Poisson law with rateλ. Service times, switch on and switch off durations are exponentially distributed with the parametersμ, α and β, respectively. The system state is described by the vector (s, k), where k is the number of customers in the system, s is the server state. Here s = 0 means that the system is in the standby mode, s=1 reflects switch-on mode and s=2 and s=3 represent operating and switch off modes, respectively. Arrival of a customer in an empty system causes change of the system state to the switch on mode. After exponentially distributed time with rate α, the system enters the operating mode, in which serving of customers is started. When the system remains empty in the operating mode, it does not switch off immediately, but waits exponentially distributed time with rate γ. If a customer arrives during that waiting period, then the system starts serving. Otherwise, the state is changed to the switch off mode. If a customer arrives during the switch off mode, then the system turns to the switch on mode immediately after the completion of the switch off. Otherwise, the system falls to the stand by mode. Figure 1 shows the transition intensities diagram.

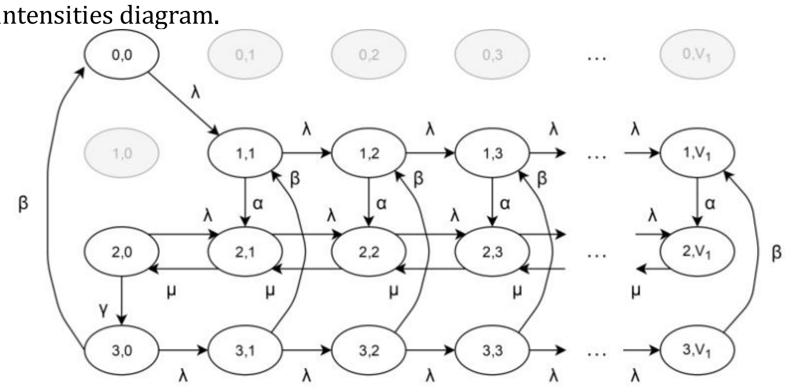

Figure. 1.Transition intensities diagram

We derive the system of equilibrium equations, based on the transition intensity diagram (Fig. 1), which makes it possible to obtain stationary probability distribution of the system:

$$p_{3,0} = \frac{\lambda}{\beta} p_{0,0}; \tag{1}$$

$$p_{3,k} = \frac{\lambda}{(\lambda+\beta)} p_{3,k-1}, \quad 1 \le k \le C-1; \tag{2}$$

$$p_{3,V_1} = \frac{\lambda}{\beta} p_{3,C-1}; \tag{3}$$

$$p_{1,1} = \frac{\lambda}{(\lambda+\alpha)} p_{0,0} + \frac{\beta}{(\lambda+\alpha)} p_{3,1}; \tag{4}$$

$$p_{1,k} = \frac{\lambda}{(\lambda+\alpha)} p_{1,k-1} + \frac{\beta}{(\lambda+\alpha)} p_{3,k}, \quad 2 \le k \le C-1; \tag{5}$$

$$p_{1,C} = \frac{\lambda}{\alpha} p_{1,C-1} + \frac{\beta}{\alpha} p_{3,C}; \tag{6}$$

$$p_{2,0} = \frac{\lambda+\beta}{\gamma} p_{3,0}; \tag{7}$$

$$p_{2,1} = \frac{\lambda+\gamma}{\mu} p_{2,0}; \tag{8}$$

$$p_{2,2}=\frac{(\lambda+\mu)}{\mu}p_{2,1}-\frac{\lambda}{\mu}p_{2,0}-\frac{\alpha}{\mu}p_{1,1}; \quad (9)$$

$$p_{2,k+1}=\frac{(\lambda+\mu)}{\mu}p_{2,k}-\frac{\lambda}{\mu}p_{2,k-1}-\frac{\alpha}{\mu}p_{1,k}, \quad 3\le k\le C-1; \quad (10)$$

$$p_{2,C}=\frac{\alpha}{\mu}p_{1,C}+\frac{\lambda}{\mu}p_{2,C-1}; \quad (11)$$

$$\sum_{i=0}^{3}\sum_{j=0}^{V_1}p_{i,j}=1. \quad (12)$$

Taking into account the normalization condition (12) and using matrix methods, the system of equations of equilibrium (1) – (11) can be solved numerically, but below we represent the analytical solution of (1) – (11).

The expression for $p_{3,0}$ follows directly from formula (1):

$$p_{3,0}=\frac{\lambda}{\beta}p_{0,0}. \quad (13)$$

The stationary probabilities $p_{3,k}$ follow from formula (2) taking into account formula (1):

$$p_{3,k}=\frac{\lambda}{\lambda+\beta}p_{3,k-1}=\left(\frac{\lambda}{\lambda+\beta}\right)^{k}p_{3,0}=\frac{\lambda^{k-1}}{(\lambda+\beta)^{k}}p_{0,0}, \quad 1\le k\le C-1. \quad (14)$$

The equation for stationary probability$p_{3,V}$ can be represented by formula (3) and expression (14):

$$p_{3,C}=\frac{\lambda}{\beta}p_{3,C-1}=\frac{\lambda}{\beta}\left(\frac{\lambda}{\lambda+\beta}\right)^{C-1}p_{3,0}=\frac{\lambda}{\beta}\left(\frac{\lambda}{\lambda+\beta}\right)^{C-1}\frac{\beta}{\lambda}p_{0,0}=\left(\frac{\lambda}{\lambda+\beta}\right)^{C-1}p_{0,0}. \quad (15)$$

The expression for $p_{1,1}$ follows from formula (4) and the expression for $p_{3,1}$, is obtained from (14):

$$p_{3,1}=\left(\frac{\lambda}{\lambda+\beta}\right)p_{3,0}=\left(\frac{\beta}{\lambda+\beta}\right)p_{0,0}$$

$$p_{1,1}=\frac{\lambda}{(\lambda+\alpha)}p_{0,0}+\frac{\beta}{(\lambda+\alpha)}p_{3,1}=\frac{1}{\lambda+\alpha}\left(\lambda p_{0,0}+\frac{\beta^2}{\lambda+\beta}p_{0,0}\right)=\frac{\lambda^2+\beta\lambda+\beta^2}{(\lambda+\alpha)(\lambda+\beta)}p_{0,0}. \quad (16)$$

The stationary probabilities $p_{1,k}$ follow from (5) by substituting expression (14) and the simple algebraic transformations:

$$p_{1,k}=\frac{\lambda}{(\lambda+\alpha)}p_{1,k-1}+\frac{\beta}{(\lambda+\alpha)}p_{3,k}, \quad 2\le k\le C-1$$

$$=\frac{\lambda}{(\lambda+\alpha)}p_{1,k-1}+\frac{\beta^2\lambda^{k-1}}{(\lambda+\beta)^{k}(\lambda+\alpha)}p_{0,0}=$$

$$=\frac{\lambda}{(\lambda+\alpha)}\left[\frac{\lambda}{(\lambda+\alpha)}p_{1,k-2}+\frac{\beta^2\lambda^{k-2}}{(\lambda+\beta)^{k-1}(\lambda+\alpha)}p_{0,0}\right]+\frac{\beta^2\lambda^{k-1}}{(\lambda+\beta)^{k}(\lambda+\alpha)}p_{0,0}=$$

$$=\left(\frac{\lambda}{\lambda+\alpha}\right)^{k-1}p_{1,1}+\sum_{i=1}^{k-1}\frac{\beta^2\lambda^{k-1}}{(\lambda+\beta)^{k+1-i}(\lambda+\alpha)^{i}}p_{0,0}=$$

$$=\left(\frac{\lambda}{\lambda+\alpha}\right)^{k-1}p_{1,1}+\frac{\beta^2\lambda^{k-1}}{(\lambda+\beta)^{k}(\lambda+\alpha)}p_{0,0}\sum_{i=1}^{k-1}\frac{(\lambda+\beta)^{i-1}}{(\lambda+\alpha)^{i-1}}=$$

$$=\left(\frac{\lambda}{\lambda+\alpha}\right)^{k-1}p_{1,1}+\frac{\beta^2\lambda^{k-1}}{(\lambda+\beta)^{k}(\lambda+\alpha)}p_{0,0}\sum_{i=1}^{k-1}\frac{1-\left(\frac{\lambda+\beta}{\lambda+\alpha}\right)^{k}}{1-\left(\frac{\lambda+\beta}{\lambda+\alpha}\right)}=$$

$$=\frac{\lambda^{k-1}(\lambda^2+\beta\lambda+\beta^2)}{(\lambda+\alpha)^{k}(\lambda+\beta)}p_{0,0}+\frac{\frac{\beta^2\lambda^{k-1}}{(\lambda+\beta)^{k}(\lambda+\alpha)}\left[\frac{(\lambda+\alpha)^{k}-(\lambda+\beta)^{k}}{(\lambda+\alpha)^{k}}\right]}{\frac{\lambda+\alpha-\lambda-\beta}{\lambda+\alpha}}p_{0,0}=$$

$$=\frac{\lambda^{k-1}(\lambda^2+\beta\lambda+\beta^2)}{(\lambda+\alpha)^{k}(\lambda+\beta)}p_{0,0}+\frac{\beta^2\lambda^{k-1}(\alpha-\beta)\left[(\lambda+\alpha)^{k}-(\lambda+\beta)^{k}\right]}{(\lambda+\beta)^{k}(\lambda+\alpha)^{k}}p_{0,0}. \quad (17)$$

The expression for $p_{1,V}$ follows from (6) by substituting expression (15) and the simple algebraic transformations:

$$p_{1,C}=\frac{\lambda}{\alpha}p_{1,C-1}+\frac{\beta}{\alpha}p_{3,C}=$$

$$=\frac{\lambda}{\alpha}\left[\frac{\lambda^{k-1}\left(\lambda^2+\beta\lambda+\beta^2\right)}{(\lambda+\alpha)^k(\lambda+\beta)}+\frac{\beta^2\lambda^{k-1}(\alpha-\beta)\left[(\lambda+\alpha)^k-(\lambda+\beta)^k\right]}{(\lambda+\beta)^k(\lambda+\alpha)^k}\right]p_{0,0}+\frac{\beta}{\alpha}\left(\frac{\lambda}{\lambda+\beta}\right)^{C-1}p_{0,0}. \tag{18}$$

The expression for calculating stationary probabilities for the operating mode $p_{2,0}$ follows from formulas (7) and (1):

$$p_{2,0}=\frac{\lambda+\beta}{\gamma}p_{3,0}=\frac{(\lambda+\beta)\lambda}{\gamma\beta}p_{0,0}. \tag{19}$$

The equation of stationary probability $p_{2,1}$ can be represented by the formula (8) and the expression (19):

$$p_{2,1}=\frac{(\lambda+\gamma)}{\mu}p_{2,0}=\frac{(\lambda+\gamma)(\lambda+\beta)\lambda}{\mu\gamma\beta}p_{0,0}. \tag{20}$$

The expression for $p_{2,2}$ is obtained by substituting expressions (16), (19) and (20) into formula (9):

$$p_{2,2}=\frac{(\lambda+\mu)}{\mu}p_{2,1}-\frac{\lambda}{\mu}p_{2,0}-\frac{\alpha}{\mu}p_{1,1}==\frac{(\lambda+\mu)(\lambda+\gamma)(\lambda+\beta)\lambda}{\mu\gamma\beta}p_{0,0}-\frac{(\lambda+\beta)\lambda}{\gamma\beta}p_{0,0}-\frac{\alpha\left(\lambda^2+\beta\lambda+\beta^2\right)}{\mu(\lambda+\alpha)(\lambda+\beta)}p_{0,0}=$$

$$=\frac{(\lambda+\beta)\lambda}{\mu\gamma\beta}\left(\frac{(\lambda+\mu)(\lambda+\gamma)}{\mu}-\lambda\right)p_{0,0}-\frac{\alpha\left(\lambda^2+\beta\lambda+\beta^2\right)}{\mu(\lambda+\alpha)(\lambda+\beta)}p_{0,0}=$$

$$=\frac{\lambda(\lambda+\beta)\left(\lambda^2+\lambda\gamma+\mu\gamma\right)}{\mu^2\gamma\beta}p_{0,0}-\frac{\alpha\left(\lambda^2+\beta\lambda+\beta^2\right)}{\mu(\lambda+\alpha)(\lambda+\beta)}p_{0,0}. \tag{21}$$

The stationary probabilities $p_{2,k}$ follow from (10) and the simple algebraic transformations:

$$(\lambda+\mu)p_{2,k}=\alpha p_{1,k}+\mu p_{2,k+1}+\lambda p_{2,k-1},\quad 2\le k\le C-1;$$

$$\mu p_{2,k+1}=(\lambda+\mu)p_{2,k}-\lambda p_{2,k-1}-\alpha p_{1,k},\quad 2\le k\le C-1;$$

$$\mu p_{2,k}=(\lambda+\mu)p_{2,k-1}-\lambda p_{2,k-2}-\alpha p_{1,k-1},\quad 3\le k\le C;$$

$$p_{2,k}=\frac{\lambda+\mu}{\mu}p_{2,k-1}-\frac{\lambda}{\mu}p_{2,k-2}-\frac{\alpha}{\mu}p_{1,k-1},\quad 3\le k\le C;$$

$$p_{2,k}=\frac{\lambda+\mu}{\mu}\left[\frac{\lambda+\mu}{\mu}p_{2,k-2}-\frac{\lambda}{\mu}p_{2,k-3}-\frac{\alpha}{\mu}p_{1,k-2}\right]-\frac{\lambda}{\mu}p_{2,k-2}-\frac{\alpha}{\mu}p_{1,k-1}=$$

$$=\left(\frac{\lambda+\mu}{\mu}\right)^2p_{2,k-2}-\frac{\lambda(\lambda+\mu)}{\mu^2}p_{2,k-3}-\frac{\alpha(\lambda+\mu)}{\mu^2}p_{1,k-2}-\frac{\lambda}{\mu}p_{2,k-2}-\frac{\alpha}{\mu}p_{1,k-1}=$$

$$=\left(\frac{(\lambda+\mu)^2}{\mu^2}-\frac{\lambda}{\mu}\right)p_{2,k-2}-\frac{\lambda(\lambda+\mu)}{\mu^2}p_{2,k-3}-\frac{\alpha}{\mu}\left[p_{1,k-1}+\frac{\lambda+\mu}{\mu^2}p_{1,k-2}\right]=$$

$$=\frac{(\lambda+\mu)^2-\lambda\mu}{\mu^2}p_{2,k-2}-\frac{\lambda(\lambda+\mu)}{\mu^2}p_{2,k-3}-\frac{\alpha}{\mu}\left[p_{1,k-1}+\frac{\lambda+\mu}{\mu^2}p_{1,k-2}\right]=$$

$$=\frac{\lambda^2+\lambda\mu+\mu^2}{\mu^2}\left[\frac{\lambda+\mu}{\mu}p_{2,k-3}-\frac{\lambda}{\mu}p_{2,k-4}-\frac{\alpha}{\mu}p_{1,k-3}\right]-\frac{\lambda(\lambda+\mu)}{\mu^2}p_{2,k-3}-\frac{\alpha}{\mu}\left[p_{1,k-1}+\frac{\lambda+\mu}{\mu^2}p_{1,k-2}\right]=$$

$$=\frac{\lambda^3+\lambda^2\mu+\lambda\mu^2+\mu^3}{\mu^3}p_{2,k-3}-\frac{\left(\lambda^2+\lambda\mu+\mu^2\right)\lambda}{\mu^3}p_{2,k-4}-$$

$$-\frac{\alpha}{\mu}\left[p_{1,k-1}+\frac{(\lambda+\mu)}{\mu^2}p_{1,k-2}+\frac{\left(\lambda^2+\lambda\mu+\mu^2\right)}{\mu^2}p_{1,k-3}\right]=$$

$$=\frac{\lambda^3+\lambda^2\mu+\lambda\mu^2+\mu^3}{\mu^3}\left[\frac{\lambda+\mu}{\mu}p_{2,k-4}-\frac{\lambda}{\mu}p_{2,k-5}-\frac{\alpha}{\mu}p_{1,k-4}\right]-\frac{\left(\lambda^2+\lambda\mu+\mu^2\right)\lambda}{\mu^3}p_{2,k-4}-$$

$$-\frac{\alpha}{\mu}\left[p_{1,k-1}+\frac{(\lambda+\mu)}{\mu^2}p_{1,k-2}+\frac{\left(\lambda^2+\lambda\mu+\mu^2\right)}{\mu^2}p_{1,k-3}\right]=$$

$$=\frac{\left(\lambda^3+\lambda^2\mu+\lambda\mu^2+\mu^3\right)(\lambda+\mu)-\left(\lambda^2+\lambda\mu+\mu^2\right)}{\mu^4}p_{2,k-4}-\frac{\left(\lambda^3+\lambda^2\mu+\lambda\mu^2+\mu^3\right)\lambda}{\mu^4}p_{2,k-5}$$

$$-\frac{\alpha}{\mu}\left[p_{1,k-1}+\frac{(\lambda+\mu)}{\mu^2}p_{1,k-2}+\frac{\left(\lambda^2+\lambda\mu+\mu^2\right)}{\mu^2}p_{1,k-3}+\frac{\left(\lambda^3+\lambda^2\mu+\lambda\mu^2+\mu^3\right)}{\mu^3}p_{1,k-4}\right]=$$

$$=\frac{\lambda^4+\lambda^3\mu+\lambda^2\mu^2+\lambda\mu^3+\mu^4}{\mu^4}p_{2,k-4}-\frac{\left(\lambda^3+\lambda^2\mu+\lambda\mu^2+\mu^3\right)\lambda}{\mu^4}p_{2,k-5} \tag{22}$$

$$-\frac{\alpha}{\mu}\left[p_{1,k-1}+\frac{(\lambda+\mu)}{\mu^2}p_{1,k-2}+\frac{\left(\lambda^2+\lambda\mu+\mu^2\right)}{\mu^2}p_{1,k-3}+\frac{\left(\lambda^3+\lambda^2\mu+\lambda\mu^2+\mu^3\right)}{\mu^3}p_{1,k-4}\right].$$

We represent the resulting expression (22) as sum of sequences:

$$p_{2,k}=\frac{\sum_{i=0}^{4}\lambda^i\mu^{4-i}}{\mu^4}p_{2,k-4}-\frac{\lambda\sum_{i=0}^{3}\lambda^i\mu^{3-i}}{\mu^4}p_{2,k-5}-$$

$$-\frac{\alpha}{\mu}\left[p_{1,k-1}+\frac{\sum_{i=0}^{1}\lambda^i\mu^{1-i}}{\mu^2}p_{1,k-2}+\frac{\sum_{i=0}^{2}\lambda^i\mu^{2-i}}{\mu^2}p_{1,k-3}+\frac{\sum_{i=0}^{3}\lambda^i\mu^{3-i}}{\mu^3}p_{1,k-4}\right]=$$

$$=\frac{\sum_{i=0}^{4}\lambda^i\mu^{4-i}}{\mu^4}p_{2,k-4}-\frac{\lambda\sum_{i=0}^{3}\lambda^i\mu^{3-i}}{\mu^4}p_{2,k-5}-\frac{\alpha}{\mu}\sum_{l=0}^{4-1}\frac{\sum_{i=0}^{l}\lambda^i\mu^{l-i}}{\mu l}p_{1,k-l-1}=$$

$$=\ldots=\frac{\sum_{i=0}^{g}\lambda^i\mu^{g-i}}{\mu^g}p_{2,k-g}-\frac{\lambda\sum_{i=0}^{g-1}\lambda^i\mu^{g-1-i}}{\mu^g}p_{2,k-g-1}-\frac{\alpha}{\mu}\sum_{l=0}^{g-1}\frac{\sum_{i=0}^{l}\lambda^i\mu^{l-i}}{\mu l}p_{1,k-l-1}=$$

$$=\{k-g=2\quad\Rightarrow\quad g=k-2\}=$$

$$=\frac{\sum_{i=0}^{k-2}\lambda^i\mu^{k-2-i}}{\mu^{k-2}}p_{2,2}-\frac{\lambda\sum_{i=0}^{k-3}\lambda^i\mu^{k-3-i}}{\mu^{k-2}}p_{2,1}-\frac{\alpha}{\mu}\sum_{l=0}^{k-3}\frac{\sum_{i=0}^{l}\lambda^i\mu^{l-i}}{\mu l}p_{1,k-l-1}=$$

$$=\frac{\sum_{i=0}^{k-2}\lambda^i\mu^{k-2-i}}{\mu^{k-2}}\left(\frac{\lambda(\lambda+\beta)(\lambda^2+\lambda\gamma+\mu\gamma)}{\mu^2\gamma\beta}-\frac{\alpha(\lambda^2+\beta\lambda+\beta^2)}{\mu(\lambda+\alpha)(\lambda+\beta)}\right)p_{0,0}-\frac{\lambda\sum_{i=0}^{k-3}\lambda^i\mu^{k-3-i}}{\mu^{k-2}}\left(\frac{(\lambda+\gamma)(\lambda+\beta)\lambda}{\mu\gamma\beta}\right)p_{0,0}$$

$$-\frac{\alpha}{\mu}\sum_{l=0}^{k-3}\frac{\sum_{i=0}^{l}\lambda^i\mu^{l-i}}{\mu^l}\left(\frac{\lambda^{k-l-2}(\lambda^2+\beta\lambda+\beta^2)}{(\lambda+\alpha)^{k-l-1}(\lambda+\beta)}+\frac{\beta^2\lambda^{k-l-2}(\alpha-\beta)\left[(\lambda+\alpha)^{k-l-1}-(\lambda+\beta)^{k-l-1}\right]}{(\lambda+\beta)^{k-l-1}(\lambda+\alpha)^{k-l-1}}\right)p_{0,0}. \tag{23}$$

The resulting expression (23) can be represented in three parts:

$$I=\frac{\sum_{i=0}^{k-2}\lambda^i\mu^{k-2-i}}{\mu^{k-2}}=\sum_{i=0}^{k-2}\lambda^i\mu^{-i}=\sum_{i=0}^{k-2}\left(\frac{\lambda}{\mu}\right)^i=\frac{1-\left(\frac{\lambda}{\mu}\right)^{k-1}}{1-\frac{\lambda}{\mu}}=\frac{1-\rho^{k-1}}{1-\rho}; \tag{24}$$

$$II=\frac{\lambda}{\mu^{k-2}}\sum_{i=0}^{k-3}\lambda^i\mu^{k-3-i}=\sum_{i=0}^{k-3}\lambda^{i+1}\mu^{-(i+1)}=\sum_{i=0}^{k-3}\left(\frac{\lambda}{\mu}\right)^{i+1}=\frac{\lambda}{\mu}\sum_{i=0}^{k-3}\left(\frac{\lambda}{\mu}\right)^i=\frac{\lambda}{\mu}\frac{1-\left(\frac{\lambda}{\mu}\right)^{k-2}}{1-\frac{\lambda}{\mu}}=\rho\frac{1-\rho^{k-2}}{1-\rho}; \tag{25}$$

$$III=\frac{\lambda}{\mu}\sum_{i=0}^{k-3}\sum_{i=0}^{l}\lambda^i\mu^{-i}p_{1,k-l-1}=\rho\sum_{i=0}^{k-3}\sum_{i=0}^{l}\rho^i p_{1,k-l-1}=\rho\sum_{l=0}^{k-3}p_{1,k-l-1}\sum_{i=0}^{l}\rho^i=$$

$$=\rho\sum_{l=0}^{k-3}p_{1,k-l-1}\frac{1-\rho^{k-2}}{1-\rho}=\frac{\rho}{1-\rho}\sum_{l=0}^{k-3}(1-\rho^{l+1})p_{1,k-l-1}=$$

$$=p_{0,0}\frac{\rho}{1-\rho}\sum_{l=0}^{k-3}(1-\rho^{l+1})\left[\frac{\lambda^{k-l-2}(\lambda^2+\beta\lambda+\beta^2)}{(\lambda+\alpha)^{k-l-1}(\lambda+\beta)}+\frac{\beta^2\lambda^{k-l-2}(\alpha-\beta)\left[(\lambda+\alpha)^{k-l-1}-(\lambda+\beta)^{k-l-1}\right]}{(\lambda+\beta)^{k-l-1}(\lambda+\alpha)^{k-l-1}}\right]=$$

$$=p_{0,0}\frac{\rho}{1-\rho}\left[\sum_{l=0}^{k-3}\frac{\lambda^{k-l-2}(\lambda^2+\beta\lambda+\beta^2)}{(\lambda+\alpha)^{k-l-1}(\lambda+\beta)}-\sum_{l=0}^{k-3}\frac{\lambda^{k-1}(\lambda^2+\beta\lambda+\beta^2)}{\mu^{l+1}(\lambda+\alpha)^{k-l-1}(\lambda+\beta)}+\right.$$

$$\left.+\sum_{l=0}^{k-3}\frac{\beta^2\lambda^{k-l-2}(\alpha-\beta)\left[(\lambda+\alpha)^{k-l-1}-(\lambda+\beta)^{k-l-1}\right]}{(\lambda+\beta)^{k-l-1}(\lambda+\alpha)^{k-l-1}}-\sum_{l=0}^{k-3}\frac{\beta^2\lambda^{k-l}(\alpha-\beta)\left[(\lambda+\alpha)^{k-l-1}-(\lambda+\beta)^{k-l-1}\right]}{\mu^{l+1}(\lambda+\beta)^{k-l-1}(\lambda+\alpha)^{k-l-1}}\right]. \tag{26}$$

In turn, we represent the third part (26) of expression (23) in the form of four parts:

$$III_1=\sum_{l=0}^{k-3}\frac{\lambda^{k-l-2}}{(\lambda+\alpha)^{k-l-1}}=\frac{\lambda^{k-2}}{(\lambda+\alpha)^{k-1}}\sum_{l=0}^{k-3}\left(\frac{\lambda+\alpha}{\lambda}\right)^l=\frac{\lambda^{k-2}}{(\lambda+\alpha)^{k-1}}\frac{1-\left(\frac{\lambda+\alpha}{\lambda}\right)^{k-2}}{1-\left(\frac{\lambda+\alpha}{\lambda}\right)}=$$

$$=-\left(\frac{\lambda}{\lambda+\alpha}\right)^{k-1}\frac{1-\left(\frac{\lambda+\alpha}{\lambda}\right)^{k-2}}{\alpha}=\frac{\lambda}{\alpha(\lambda+\alpha)}\left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{k-2}\right]; \tag{27}$$

$$III_2=\sum_{l=0}^{k-3}\frac{1}{\mu^{l-1}(\lambda+\alpha)^{k-l-1}}=\frac{1}{(\lambda+\alpha)^{k-1}\mu}\sum_{l=0}^{k-3}\left(\frac{\lambda+\alpha}{\mu}\right)^l=\frac{1}{(\lambda+\alpha)^{k-1}\mu}\frac{1-\left(\frac{\lambda+\alpha}{\mu}\right)^{k-1}}{1-\frac{\lambda+\alpha}{\mu}}; \tag{28}$$

$$III_3=\sum_{l=0}^{k-3}\frac{\lambda^{k-l-2}\left[(\lambda+\alpha)^{k-l-1}-(\lambda+\beta)^{k-l-1}\right]}{(\lambda+\beta)^{k-l-1}(\lambda+\alpha)^{k-l-1}}=\sum_{l=0}^{k-3}\frac{\lambda^{k-l-2}}{(\lambda+\beta)^{k-l-1}}-\sum_{l=0}^{k-3}\frac{\lambda^{k-l-2}}{(\lambda+\alpha)^{k-l-1}}=$$

$$=\frac{\lambda^{k-2}}{(\lambda+\beta)^{k-1}}\sum_{l=0}^{k-3}\left(\frac{\lambda+\beta}{\lambda}\right)^{l}-\frac{\lambda^{k-2}}{(\lambda+\alpha)^{k-1}}\sum_{l=0}^{k-3}\left(\frac{\lambda+\alpha}{\lambda}\right)^{l}=$$

$$=\frac{\lambda^{k-2}}{(\lambda+\beta)^{k-1}}\cdot\frac{1-\left(\frac{\lambda+\beta}{\lambda}\right)^{k-2}}{1-\left(\frac{\lambda+\beta}{\lambda}\right)}-\frac{\lambda^{k-2}}{(\lambda+\alpha)^{k-1}}\cdot\frac{1-\left(\frac{\lambda+\alpha}{\lambda}\right)^{k-2}}{1-\left(\frac{\lambda+\alpha}{\lambda}\right)}=$$

$$=\left(\frac{\lambda}{\lambda+\beta}\right)^{k-1}\cdot\frac{\left(\frac{\lambda+\beta}{\lambda}\right)^{k-2}-1}{\beta}-\left(\frac{\lambda}{\lambda+\alpha}\right)^{k-1}\cdot\frac{\left(\frac{\lambda+\alpha}{\lambda}\right)^{k-2}-1}{\alpha}; \qquad (29)$$

$$III_4=\sum_{l=0}^{k-3}\frac{\left[(\lambda+\alpha)^{k-l-1}-(\lambda+\beta)^{k-l-1}\right]}{\mu^{l+1}(\lambda+\beta)^{k-l-1}(\lambda+\alpha)^{k-l-1}}=$$

$$=\sum_{l=0}^{k-3}\frac{(\lambda+\alpha)^{k-l-1}}{\mu^{l+1}(\lambda+\beta)^{k-l-1}(\lambda+\alpha)^{k-l-1}}-\sum_{l=0}^{k-3}\frac{(\lambda+\beta)^{k-l-1}}{\mu^{l+1}(\lambda+\beta)^{k-l-1}(\lambda+\alpha)^{k-l-1}}=$$

$$=\frac{1}{\mu(\lambda+\beta)^{k-1}}\sum_{l=0}^{k-3}\left(\frac{\lambda+\beta}{\mu}\right)^{l}-\frac{1}{\mu(\lambda+\alpha)^{k-1}}\sum_{l=0}^{k-3}\left(\frac{\lambda+\alpha}{\mu}\right)^{l}=$$

$$=\frac{1}{\mu(\lambda+\beta)^{k-1}}\cdot\frac{1-\left(\frac{\lambda+\beta}{\mu}\right)^{k-2}}{1-\left(\frac{\lambda+\beta}{\mu}\right)}-\frac{1}{\mu(\lambda+\alpha)^{k-1}}\cdot\frac{1-\left(\frac{\lambda+\alpha}{\mu}\right)^{k-2}}{1-\left(\frac{\lambda+\alpha}{\mu}\right)}. \qquad (30)$$

Then we substitute the obtained expressions (27) – (30) into expression (26):

$$III=p_{0,0}\frac{\rho}{1-\rho}\left[\frac{\left(\lambda^2+\beta\lambda+\beta^2\right)}{(\lambda+\beta)}\cdot\frac{\lambda}{\alpha(\lambda+\alpha)}\left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{k-2}\right]-\right.$$

$$-\frac{\lambda^{k-1}\left(\lambda^2+\beta\lambda+\beta^2\right)}{(\lambda+\beta)}\frac{1}{(\lambda+\alpha)^{k-1}\mu}\frac{1-\left(\frac{\lambda+\alpha}{\mu}\right)^{k-1}}{1-\frac{\lambda+\alpha}{\mu}}+$$

$$+\beta^2(\alpha-\beta)\left[\left(\frac{\lambda}{\lambda+\beta}\right)^{k-1}\cdot\frac{\left(\frac{\lambda+\beta}{\lambda}\right)^{k-2}-1}{\beta}-\left(\frac{\lambda}{\lambda+\alpha}\right)^{k-1}\cdot\frac{\left(\frac{\lambda+\alpha}{\lambda}\right)^{k-2}-1}{\alpha}\right]-$$

$$\left.+\frac{\beta^2(\alpha-\beta)}{\mu}\cdot\left[\frac{\lambda^{k-1}}{(\lambda+\beta)^{k-1}}\cdot\frac{1-\left(\frac{\lambda+\beta}{\mu}\right)^{k-2}}{1-\left(\frac{\lambda+\beta}{\mu}\right)}-\frac{\lambda^{k-1}}{(\lambda+\alpha)^{k-1}}\cdot\frac{1-\left(\frac{\lambda+\alpha}{\mu}\right)^{k-2}}{1-\left(\frac{\lambda+\alpha}{\mu}\right)}\right]\right]$$

$$=p_{0,0}\frac{\rho}{1-\rho}\left[\frac{\left(\lambda^2+\beta\lambda+\beta^2\right)}{(\lambda+\beta)}\cdot\left[\frac{\lambda}{\alpha(\lambda+\alpha)}\left(1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{k-2}\right)-\frac{\lambda^{k-1}}{(\lambda+\beta)\mu}\frac{1-\left(\frac{\lambda+\alpha}{\mu}\right)^{k-1}}{1-\frac{\lambda+\alpha}{\mu}}\right]+\right.$$

$$+\beta^2(\alpha-\beta)\left[\left(\frac{\lambda}{\lambda+\beta}\right)^{k-1}\cdot\left(\frac{\left(\frac{\lambda+\beta}{\lambda}\right)^{k-2}-1}{\beta}-\cdot\frac{1-\left(\frac{\lambda+\beta}{\mu}\right)^{k-2}}{\mu\left(1-\frac{\lambda+\beta}{\mu}\right)}\right)-\right.$$

$$\left.\left.\left(\frac{\lambda}{\lambda+\alpha}\right)^{k-1}\cdot\left(\frac{\left(\frac{\lambda+\alpha}{\lambda}\right)^{k-2}-1}{\alpha}-\cdot\frac{1-\left(\frac{\lambda+\alpha}{\mu}\right)^{k-2}}{\mu\left(1-\frac{\lambda+\alpha}{\mu}\right)}\right)\right]\right]. \qquad (31)$$

Then we substitute the expressions (31), (24) and (25) into the formula (23):

$$p_{2,k}=\frac{1-\rho^{k-1}}{1-\rho}\cdot\left(\frac{\lambda(\lambda+\beta)\left(\lambda^2+\lambda\gamma+\mu\gamma\right)}{\mu^2\gamma\beta}-\frac{\alpha\left(\lambda^2+\beta\lambda+\beta^2\right)}{\mu(\lambda+\alpha)(\lambda+\beta)}\right)p_{0,0}-\rho\frac{1-\rho^{k-2}}{1-\rho}\left(\frac{(\lambda+\gamma)(\lambda+\beta)\lambda}{\mu\gamma\beta}\right)p_{0,0}-$$

$$- p_{0,0}\left(\frac{\rho}{1-\rho}\right)\frac{\alpha}{\mu}\Bigg[\frac{(\lambda^2+\beta\lambda+\beta^2)}{(\lambda+\beta)}\cdot\left[\frac{\lambda}{\alpha(\lambda+\alpha)}\left(1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{k-2}\right)-\frac{\lambda^{k-1}}{(\lambda+\beta)\mu}\,\frac{1-\left(\frac{\lambda+\alpha}{\mu}\right)^{k-1}}{1-\frac{\lambda+\alpha}{\mu}}\right]+$$

$$+\beta^2(\alpha-\beta)\left[\left(\frac{\lambda}{\lambda+\beta}\right)^{k-1}\cdot\left(\frac{\left(\frac{\lambda+\beta}{\lambda}\right)^{k-2}-1}{\beta}-\cdot\frac{1-\left(\frac{\lambda+\beta}{\mu}\right)^{k-2}}{\mu\left(1-\frac{\lambda+\beta}{\mu}\right)}\right)-\left(\frac{\lambda}{\lambda+\alpha}\right)^{k-1}\cdot\left(\frac{\left(\frac{\lambda+\alpha}{\lambda}\right)^{k-2}-1}{\alpha}-\cdot\frac{1-\left(\frac{\lambda+\alpha}{\mu}\right)^{k-2}}{\mu\left(1-\frac{\lambda+\alpha}{\mu}\right)}\right)\right]\Bigg] \quad (32)$$

3. Energy consumption indicators

After receiving the system stationary distribution, we calculate the energy consumption indicators. We will assume that in the switch on / off mode, the power consumption is constant and equal to the average value. In the operating mode, the power consumption depends on the server occupancy. By analogy with the formula given in [3,4,5], we derive the formula for the average server power consumption:

$$P = P_0\sum_{i=0}^{C} p_{0,i} + P_1\sum_{i=0}^{C} p_{1,i} + P_3\sum_{i=0}^{C} p_{3,i} + \sum_{i=0}^{C} P_{2,i}\,p_{2,i}, \quad (33)$$

where

$$P_{2,k} = P_{2,\min} + \frac{P_{2,\max} - P_{2,\min}}{V_1}k. \quad (34)$$

The values of P_i were taken from [2], according to which P_0 = 10 W, P_1 = 170 W, P_3 = 120 W,$P_{2,\,min}$ = 105 W and $P_{2,\,max}$ = 268 W.

The average number N of customers in the system is equal to the average effective arrival rate $\lambda(1\text{-}\pi)$ multiplied by the average sojourn time T. Expressed algebraically the law is

$$N = \lambda(1-\pi)T, \quad (35)$$

where blocking probability π is

$$\pi = p_{1,C} + p_{2,C} + p_{3,C}. \quad (36)$$

The average number N of customers is given by

$$N = \sum_{k=0}^{3}\sum_{i=1}^{C} i p_{k,i} \quad (37)$$

The average sojourn time T follows directly from formulas (35) and (37):

$$T = \frac{\sum_{k=0}^{3}\sum_{i=1}^{C} i p_{k,i}}{\lambda(1-\pi)}. \quad (38)$$

4. Numeral analysis

The results of numerical analysis for the values C=*7*, μ=*20,* α=*1,* β=*2* are presented in figures 2 – 4.

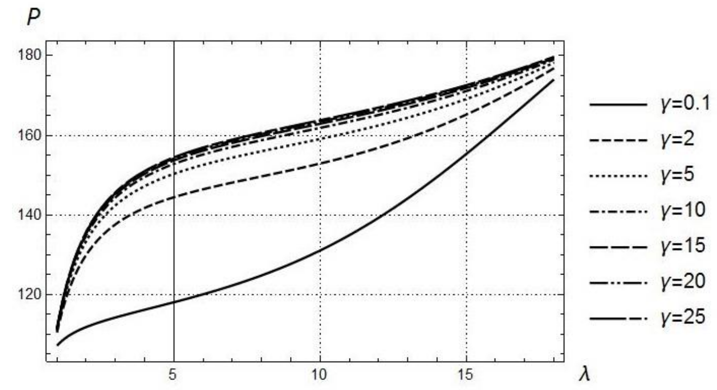

Figure. 2. The dependence of the power consumption P on the arrival flow intensity λ

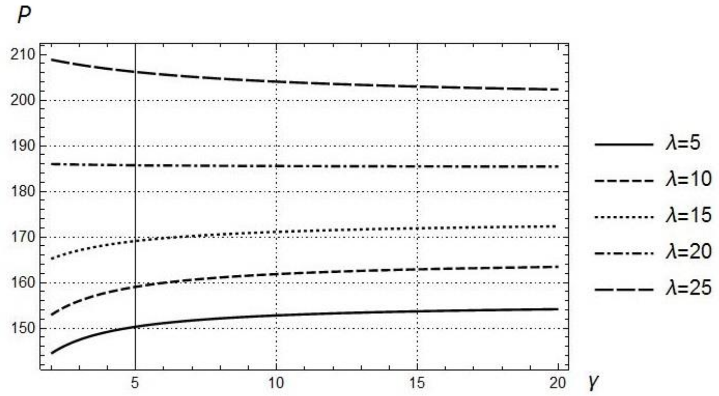

Figure. 3. The dependence of the power consumption P on the waiting time before the system goes to the standby mode

The plots of the server's power consumption (fig. 2) for our model show that the consumed power increases very fast for small values of the arrival flow intensity λ, also note that with the increase of waiting time, during which the system doesn't go into standby mode, the power consumption also increases.

In Fig. 3, we note that the largest drop for power consumption occurs at small values of γ that corresponds to large values of waiting time before the system goes to the standby mode, and that with the increase of the load intensity λ, energy consumption also increases.

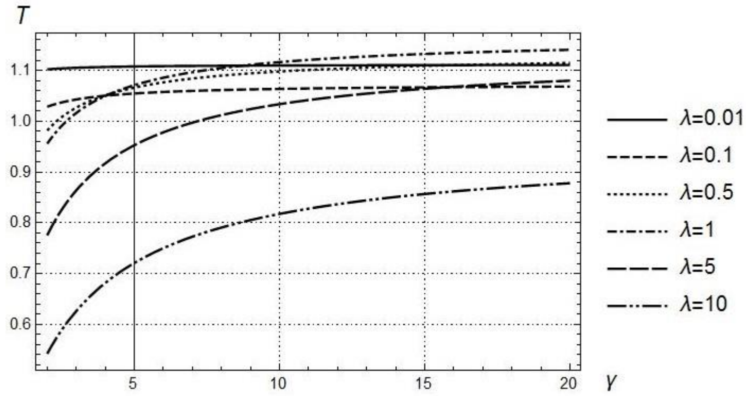

Figure. 4. The dependence of the average time T on the waiting time before the system goes to the standby mode

In Fig. 4, we note that at small values of γ difference in sojourn time is not big. With increase of γ, the average sojourn time also increases. It means, the larger the waiting time before the system goes to the standby mode, the greater sojourn time.

5. Conclusion

In the paper, we considered a cloud computing system in which the server switches off after a random time after it was left empty. The intensity γ monotonically affects the average consumed power, so we need to investigate its effect on performance measures also.

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Note on the authors:

Sopin Eduard S., professor at the Applied Probability and Informatics departmentof Peoples' Friendship University of Russia; PhD, senior researcher at the Institute of Informatics Problems, Federal Research Center Computer Science and Control of the Russian Academy of Sciences, sopin_es@rudn.university

Daraseliya Anastasia V., student of Applied Probability and Informatics department, Peoples' Friendship University of Russia, nastyadar6@gmail.com

Об авторах:

Сопин Эдуард Сергеевич, кандидат физико-математических наук, доцент кафедры прикладной информатики и теории вероятностей, Российский университет дружбы народов; старший научный сотрудник Института прикладной информатики, Федеральный исследовательский центр «Информатика и управление» Российской академии наук, sopin_es@rudn.university

Дараселия Анастасия Валерьевна, студент кафедры прикладной информатики и теории вероятностей, Российский университет дружбы народов, nastyadar6@gmail.com