Propaganda Battle with Two-Component Agenda*

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Abstract. A propaganda battle is considered in which each party allocates their broadcasting resources between two topics. According to the agenda-setting theory, the public attention on each of these topics depends on the proportion of broadcasting that it gets from both parties. On each of the topics, each member of the population supports one of the parties on the basis of the individual's attitudes (that is, the individual's topic-relevant predisposition), the propaganda broadcasting flows from both parties and the opinions of other members of the population. The individual's party attachment is determined by their positions on both the topics with the account of relative saliences of these topics. Over the course of time they may shift their support (on any topic or both) to the other party under the influence of belligerent parties' broadcasting and the opinions of other individuals. Theoretically, they may switch their partisanship back and forth an unlimited number of times. As a consequence, the number of supporters for each party varies over time. In this framework, we introduce the Blotto game in which each of the parties allocates its broadcasting resource between the topics and the payoff is the difference between the belligerent parties' numbers of supporters at the end of the battle.

Keywords: propaganda battle, mathematical model, decision-making, agenda-setting, Blotto game.

1 Introduction

Politics today is unthinkable without propaganda battles. Fake news, social bots, allegations, personal data collecting and mining and so on. Investigations about all these things have been an amusing topic for the public and a surging area of research. This paper aims to add to this pertaining literature by analyzing a theoretical model of a propaganda battle with a two-component agenda.

It can be imagined, for instance, as an election campaign in which two main controversial topics (say, security and the performance of economy) inform the political discussion. There are two parties in the race: Right (R) and Left (L), and they oppose each other on each topic. Each party broadcasts its messages via affiliated mass media. Supporters get the messages and spread them among other individuals through interpersonal communications. At any given moment of time, each member of the population supports one of the parties and disapproves of the other. Over the course of time, they may toggle their support to the other party under the influence of media propaganda and the opinions of collocutors. As a result, the number of supporters for each party changes over time. This process is called propaganda battle.

In this paper, we consider it using our model developed in [1]. More specifically, here we introduce the ideas of the agenda-setting theory (see, for example, [2–4]) into this model. The notion of agenda can be illustrated by the example of the US presidential election in 2004. Consider the data on issue frequencies appearing in press releases of the candidates: The Republican incumbent President, G. Bush and the democratic challenger, J. Kerry(the data were presented by Tedesco in [5]). By aggregating 11 issues related to security (Homeland security, 9/11, National security, National defense, Terrorism, War on Terror, War in Iraq, Iraq, Foreign policy, Troops, Patriot Act) to inform the security topic, and two issues (economy, jobs) relating to the topic on the performance of economy, we can easily see that Bush tried to push the topic of security into the agenda more heavily than J. Kerry, whereas Kerry pushed the topic of economy (Table 1).The CNN exit poll [6] confirmed that the candidates did right in their policies: from those voters who believed terrorism to be the most important issue of the campaign, 86% voted for Bush (vs 14% for Kerry), whereas 80% of those who reckoned the most important issue is economy/jobs, voted for Kerry (vs 18% for Bush).

Table 1. Press release topic frequencies (% of the candidate's releases)

	Bush	Kerry
Security	18.3	12.2
The performance of economy	24.5	35.7

The example illustrates that during elections the parties try, in competition with each other, to shape the agenda. The propaganda battle in this example can be described as a struggle for not only how positively or negatively Bush's achievements should be assessed on each of these issues during his first presidential term but also a struggle for which topic will attract more attention from voters and will gain higher priority to them. This is the practical meaning of the agenda-setting theory.

Thus, one of the aspects of the propaganda battle is the tug-of-war of setting the agenda. From the perspective of mathematical modeling, this can be interpreted as a kind of multidimensionality of the information space, and its components can't be described as independent of each other. In other words, the problem of propaganda battle warfare on two topics cannot be reduced to two separate "one-dimensional" problems. This contrasts with all known models of information warfare, which are "one-dimensional" in this respect. We assume that each of the parties pursues its own strategy in the propaganda battle, distributing its broadcasting potential between two components of agenda, that is between two topics. The outcome of the battle depends on the constellation of belligerent parties' strategies; such situations are generally considered as game-theoretical. The specific kind of games that appears here is known as Blotto game.

In this paper, this range of ideas is integrated into the basic model of the position selection by individuals for the case of two discussed topics. Thus, each individual here is described by their attitudes (formed by the "predispositions" to support one or another point of view developed before the commencement of the propaganda battle) on two questions defining the 2-dimensional coordinate system. With regard to the previous example, these attitudes have the meaning of a long-term voter predisposition to support the security policy of Republicans or Democrats (the first coordinate) and the economic policies of Republicans or Democrats (the second coordinate) before the beginning of the election campaign. Accordingly, society as a whole is described by a two-dimensional distribution of individuals.

In what follows, Section 2 deals with a brief overview of the subject, Section 3 is devoted to building the model. It is followed by Section 4, which illustrates the application of the model for the simplest case. Section 5 is devoted to the Blotto game, Section 6 concludes.

2 A Brief Glance at the Literature

The area of research of informational dissemination in a population by means of mathematical modeling is relatively new, although the earliest model to study a single-rumor was introduced by Daley and Kendall [7] as long ago as 1964.The first model of two competing antagonistic rumors was proposed by Osei and Thompson in 1977 [8]. By the word "antagonistic", we mean that each individual can approve and relay to collocutors another one of these rumors. The earliest model describing the dissemination of information (by a single party, without any competition)via both rumors and mass-media, was proposed in book by Samarskii and Mikhailov [9].

The model of information warfare was introduced by Mikhailov and Marevtseva [10]. It considers the process in which two belligerent parties spread antagonistic messages through their affiliated media, and members of the population relay these messages to one another through interpersonal communication. In that paper, the so-called victory condition has been derived, i.e., the relation between the parameters that determines which party gets the majority of supporters when $t \to \infty$.

The mentioned models gave rise to significant literature in which society is viewed as a "continuous mass" (in contrast to considering individuals as "discrete particles"). Accordingly, equations of these models operate with "macroscopic" variables (such as, the number of information carriers) and assume homogeneous communications. Alternative approaches, which yielded hundreds of papers, include emphasis on agent-based and game theory based models and social networks [11, 12].

Within each of mentioned approaches, individuals are presented as a medium for the dissemination of information, i.e., these approaches don't consider the process of individual's decision-making on which of the belligerent parties to support. In contrast to this, the model of position selection by individuals during the propaganda battle [1] puts this process in the focus of the research. This is the model we build on here. As one of the model's variations we present here directly lead to the Blotto game, it should be also mentioned that this type of games has its own story of applications to analysis of political competition (see, for example, Merolla et al. [13,14]). Related areas include also studies of search queries [15-17] and opinion dynamic models [18, 19].

3 Model

This model is an extension of the model of position selection by individuals during propaganda battle in a population [1, 20] to the two-dimensional case. Accordingly, in this text we restrict ourselves to presenting some aspects of the model in a brief manner, focusing on new issues related to the extension to the two-dimensional space of individual's attitudes.

Consider two parties, L (left) and R (right) engaged in a propaganda battle in a population of size N_0. Two topics are discussed on which these parties have opposing positions and broadcast these positions by affiliated media. The positions are then relayed by the members of the population to each other, thereby the process of propaganda battle takes place.

Each member of the population at each moment of time has a certain latent position on each topic, which is the sum of a permanent (during a given confrontation) attitude and a dynamic component. Attitude $\varphi_i \in (-\infty, \infty)$ (where *i* is the topic number) is a tendency to support one party or another: it is formed during the previous social experience of the individual, takes into account their social position and is assumed to be unchanged throughout this confrontation. The dynamic component $\psi_i(t) \in (-\infty, \infty)$ has the meaning of a shift in the stimuli towards the support of the party R, which is determined by the social environment. It is influenced by the propaganda broadcasting, as well as mouth-to-mouth exchanges.

Attitudes φ_1, φ_2 are individual-specific, whereas dynamic components $\psi_1(t), \psi_2(t)$ characterize the information field of the population as a whole. (At the same time, more complex models can take into account that, for example, conservatives read more conservative newspapers, and liberals read more liberal newspapers. The model analyzed in [21] takes into account that some part of the population doesn't use the media at all and receives information only through interpersonal communications).

For each topic, the negative values of the latent position correspond to the support of the Left party, the positive values refer to the support of the Right party, and the greater the absolute value of $\varphi_i + \psi_i(t)$ is, the stronger the support is.

In our one-dimensional model (i.e., the model in which it is assumed that only one topic [1] is discussed in a population), the support of a particular party by some specific individual means that in interpersonal communication, the individual speaks in support of this party, thereby creating informational stimuli for other individuals. If two topics are discussed, then a situation is possible in which the individual supports one of the parties on the first topic, and the opposite party on the other; for example, $\varphi_1+\psi_1>0$, $\varphi_2+\psi_2<0$.

In this case, we put that the individual is a supporter of the Right party, if $g(\varphi_1+\psi_1)+(1-g)(\varphi_2+\psi_2)>0$, and they support the Left party in the case of the opposite inequality. Here, $g\in(0;1)$. The vector $\{g,1-g\}$ will be referred to as the agenda, it characterizes the significance of the topics in comparison with each other. Let, for example, the latent position of a certain individual in favor of party R on the first topic be equal to their latent position in favor of party L on the second topic, i.e., $\varphi_1+\psi_1=-(\varphi_2+\psi_2)$. In this case, the individual is a supporter of the party R if and only if the first topic is more significant, i.e., $g>1/2$.

Let us denote by $N(\varphi_1,\varphi_2)$ the distribution of attitudes among individuals (this value is a generalization of the one-dimensional distribution employed in one-dimensional model [1]), here

$$\iint_{\mathbb{R}^2} N(\varphi_1,\varphi_2)\,d\varphi_1 d\varphi_2 = N_0 .$$

Then, in accordance with the introduced above, the numbers of parties' R, L supporters are equal respectively

$$R=\iint_{g(\varphi_1+\psi_1)+(1-g)(\varphi_2+\psi_2)>0} N(\varphi_1,\varphi_2)\,d\varphi_1 d\varphi_2 ,$$

$$L=\iint_{g(\varphi_1+\psi_1)+(1-g)(\varphi_2+\psi_2)<0} N(\varphi_1,\varphi_2)\,d\varphi_1 d\varphi_2 .$$

(Note that the approach adopted by us contains a certain simplification: despite the fact that the agenda in the social sciences is considered common to all population, in reality it reflects a certain "average" significance of those. In other words, in reality, each individual has their own ideas about which topic is more important.)

Further, if a certain member of a population is a supporter of a certain party, but supports its position only on one topic, then we assume that when communicating with other individuals, they agitate for this party only on this topic. If she supports the party on both topics, she agitates for it on both topics. For example, an individual agitates for the party R on the first topic, if two inequalities are fulfilled simultaneously: $g(\varphi_1+\psi_1)+(1-g)(\varphi_2+\psi_2)>0$ (supports party R) and $\varphi_1+\psi_1>0$ (supports the party R on the first topic).

Thus, the numbers of individuals agitating on the first topic for the right and left parties are equal, respectively,

$$\iint_{R1} N(\varphi_1,\varphi_2)\,d\varphi_1 d\varphi_2; \quad \iint_{L1} N(\varphi_1,\varphi_2)\,d\varphi_1 d\varphi_2 ,$$

where

$$\text{R1: } g(\varphi_1+\psi_1)+(1-g)(\varphi_2+\psi_2)>0,\ \ \varphi_1+\psi_1>0, \qquad (1)$$

$$\text{L1}: \; g(\varphi_1+\psi_1)+(1-g)(\varphi_2+\psi_2)<0, \;\; \varphi_1+\psi_1<0. \tag{2}$$

Similarly, the numbers of individuals agitating on the second topic for the right and left parties are equal, respectively,

$$\iint\limits_{R2} N(\varphi_1,\varphi_2)\,d\varphi_1 d\varphi_2; \quad \iint\limits_{L2} N(\varphi_1,\varphi_2)\,d\varphi_1 d\varphi_2 ,$$

where

$$\text{R2}: \; g(\varphi_1+\psi_1)+(1-g)(\varphi_2+\psi_2)>0, \;\; \varphi_2+\psi_2>0, \tag{3}$$

$$L2: \; g(\varphi_1+\psi_1)+(1-g)(\varphi_2+\psi_2)<0, \;\; \varphi_2+\psi_2<0. \tag{4}$$

According to the agenda-setting theory [2–5], the value of g is determined by the media broadcasting: the more a certain topic is covered, the higher its share in the agenda is. In accordance with this, suppose that if b_{iL} , b_{iR} are the intensities of broadcasting of the left and right parties on the *i*-th issue, then the equation for the function $g(t)$ is

$$\frac{dg}{dt}=F(b_{1L}+b_{1R}, b_{2L}-b_{2R}, g). \tag{5}$$

As a relatively simple function $F(b_L, b_R, g)$ with desirable properties, we choose $F(b_1, b_2, g)=kg(1-g)\left[b_1/(b_1+b_2)-g\right]$. Then equation (5) takes the form

$$\frac{dg}{dt}=kg(1-g)\left[\frac{b_{1L}+b_{1R}}{b_{1L}+b_{1R}+b_{2L}+b_{2R}}-g\right]$$

If $b_{1L}+b_{1R}>b_{2L}+b_{2R}$ (i.e., the total broadcasting of both parties on the first topic is stronger than on the second one), then $g_0=(b_{1L}+b_{1R})/(b_{1L}+b_{1R}+b_{2L}+b_{2R})>0{,}5$. In other words, the more dominant the first topic is, the higher the value of $g_0 \in (0;1)$ is.

At each point in time, each individual supports the stance of one of the parties on each topic (a single person may support the Left party's stance on one of the topics and the Right party's stance on the other). The description of the decision-making mechanism is based on the Rashevsky's neurological scheme [22] , which describes the formation of an individual's reaction in response to incoming stimuli with regard to their attitude. With regard to the topic of the propaganda battle, the reaction is the individual's manifested position, i.e., their participation in the dissemination of information in support of one of the parties (on one or the both topics). Stimuli are the pieces of information that come to them (both from collocutors and from the media).

The formation of the reaction can be very roughly described as follows. Suppose one day an individual received information from three party R supporters and one party L supporter, read one newspaper article in favor of party R and two articles in favor of party L. Having weighed these stimuli in a certain way, we get a change in her position in favor of that or another party (the weight coefficients are set exogenously in this model). For example, given her attitude, she could become a more or less radical supporter of her current party or shift her support to the other party. This mechanism is described in more detail for the one-dimensional case in [1] (formal and rigorous mathematical derivation) and [20] (less formal).

In the case of a two-component agenda, in accordance with the above provisions, the model has the form

$$\frac{d\psi_1}{dt} = -a\psi_1 + b_{1R} - b_{1L} + gC\left[\iint\limits_{R1} N(\varphi_1,\varphi_2)\,d\varphi_1 d\varphi_2 - \iint\limits_{L1} N(\varphi_1,\varphi_2)\,d\varphi_1 d\varphi_2\right]; \quad (6)$$

$$\frac{d\psi_2}{dt} = -a\psi_2 + b_{2R} - b_{2L} + \\ +(1-g)C\left[\iint\limits_{R2} N(\varphi_1,\varphi_2)\,d\varphi_1 d\varphi_2 - \iint\limits_{L2} N(\varphi_1,\varphi_2)\,d\varphi_1 d\varphi_2\right]; \quad (7)$$

$$\frac{dg}{dt} = kg(1-g)\left[\frac{b_{1L}+b_{1R}}{b_{1L}+b_{1R}+b_{2L}+b_{2R}} - g\right]. \quad (8)$$

The next section is devoted to the analysis of this model in one of the simplest but informative cases, which we will call the base case.

4 The Simplest Case

The substantive question of this section is as follows. Suppose, for example, that our population is prone to the rapid dissemination of rumors (parameter C is great enough). In this case, the promising strategy is to try to gain the majority of supporters at the very beginning of the campaign. These supporters will essentially become our active agitators. This advantage in the number of agitators will allow us to increase the number of supporters throughout the remainder of the campaign. The reverse situation is also possible. Suppose that members of the population quickly lose their interest in our message (the parameter a is great enough). In this case, the initial advantage will not benefit in the long run.

Therefore, this is the question: under given parameters, if one of the parties has some advantage in the number of supporters at the initial moment of the process, will it increase over time or, on the contrary, decrease?

To deal with this question, consider the model in the simplest case, which allows to study the equations by analytical techniques. This simplest case is that the belligerent parties broadcast with equal intensities on each topic:

$$b_{1R} = b_{1L};\; b_{2R} = b_{2L},$$

and the distribution of individuals in attitudes is uniform inside the square, i.e.,

$$N(\varphi_1,\varphi_2)=\begin{cases} N_0/\left(4M^2\right), & |\varphi_1|\le M,\ |\varphi_2|\le M \\ 0, & |\varphi_1|>M \quad or \quad |\varphi_2|>M \end{cases}.$$

After some cumbersome mathematics, we get that in this case the model has the stationary solution $\psi_1=\psi_2=0$, and it is asymptotically stable if $\lambda=-a+\left(CN_0/2M\right)f\left(g_0\right)$ is negative. Here the function $f\left(g_0\right)$ is given by the formula

$$f\left(g_0\right)=\frac{2\left(1-g_0\right)^2+g_0\pm\sqrt{4\left(1-g_0\right)^3\left(2-g_0\right)+g_0^2}}{2\left(1-g_0\right)}$$

The graph of the function $f\left(g_0\right)$ (if we take the sign "+") is shown in Fig.1.

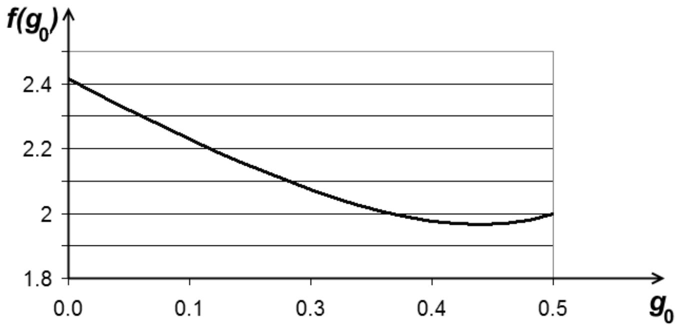

Fig. 1. The graph of the function $f\left(g_0\right)$

In accordance with this, the following conclusions take place.

1. Stability of the equilibrium state (in which parties have an equal number of supporters when $t\to\infty$) is facilitated by a low intensity of information dissemination through interpersonal communications and a high relaxation rate. This relationship can be explained as follows. Suppose that the Left party has an initial advantage, that is $L(0)>R(0)$. Strengthening this advantage is contributed by the fact that the existing majority generates a greater flow of information during communication, with the result that an individual on average receives more stimuli to join this side. On the other hand, the weakening this advantage is contributed by relaxation. These multidirectional, competing processes are characterized, respectively, by the parameters C, *a*. The advance is maintained by the high values of the first parameter and the low values of the second one. In this case, the initial advantage is crucial for the outcome: $L(\infty)>R(\infty)$. In the opposite situation (when the parameter C is sufficiently small and/or the parameter a is great enough), the initial advantage levels off and $L(\infty)=R(\infty)$.

2.The stability of the zero equilibrium is contributed by the high fuzziness of the attitudes. Namely, if on the whole, individuals have attitudes close to neutral (low values of M), then this creates a greater instability compared to the situation when the attitudes are distributed over a wide range (high values of M).

3. On the whole, approximately equal coverage of two topics by the media (i.e., values of g_0 are close to 1/2) is more favorable to the stability of zero equilibrium than a situation in which one of the topics is covered more significantly than the other (i.e. $g \approx 0$). However, this dependence isn't strictly monotonic).

The first and the second conclusions also take place for the model in which it is assumed that only one topic is discussed (see [1, 12]). The third conclusion occurs only in the transition to the two-dimensional model considered in this paper.

5 The Blotto Game

In this section, we consider the game-theoretic formulation of the problem mentioned in Section 1. In particular, each of the parties allocates their potential for broadcasting between two topics. For simplicity, suppose that possible strategies are described by integers. The result of the campaign depends on a combination of strategies. Thus, a Blotto game emerges, in which the payoffs are determined by solving the model equations.

This section is devoted to the numerical construction of the payoff matrix for the example of the situation in which a population is consolidated on the first topic and polarized on the second one. In other words, it has a normal distribution of the variable φ_1 and a mixture of normal distributions of the variable φ_2:

$$N(\varphi_1,\varphi_2) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(-\frac{\varphi_1^2}{2\sigma^2}\right)} \left[\frac{1}{2\sigma\sqrt{2\pi}} \exp\left(-\frac{(\varphi_2 - p)^2}{2\sigma^2}\right) + \frac{1}{2\sigma\sqrt{2\pi}} \exp\left(-\frac{(\varphi_2 + p)^2}{2\sigma^2}\right)\right].$$

Here the parameter p characterizes the degree of polarization of the population. The greater this parameter is the more polarized on the second topic the population is. The parameter σ characterizes, in a certain sense, the degree of homogeneity of the support for the selected party on the both topics. If this parameter is high, then the degree of support level dispersion is high; with a small parameter, individuals support the selected party approximately equally. For our numerical example, we use the following values of parameters: $a=1$, $C=1$, $p=3$, $k=1$, $\sigma=2$,and initial conditions $\varphi_1(0)=\varphi_2(0)=0$, $g(0)=0.5$.

Consider the case when one of the parties has an advantage in the possibilities of propaganda broadcasting. Let $b_{1L}+b_{2L}=6$, $b_{1R}+b_{2R}=5$. Then the Left party has seven possible strategies: to make $b_{1L}=0, b_{2L}=6$ (we will denote such a strategy as "0+6"), $b_{1L}=1, b_{2L}=5$ ("1+5" strategy) and so on. Similarly, there are six possible strategies for the Right party. For each pair of strategies, we calculated numerically the outcome of the propaganda battle (Table 1). There is the saddle point at $b_{1L}+b_{2L}=6+0$, $b_{1R}+b_{2R}=0+5$ with payoff $L(t)-R(t)=0.40$ (as $t\to\infty$).So the game is solved in pure strategies.

Table2. The outcome of the battle: $L(t)-R(t)$, $t \to \infty$.

		$b_{1R}+b_{2R}$					
		0+5	1+4	2+3	3+2	4+1	5+0
	0+6	0.16	0.32	0.46	0.52	0.49	0.33
	1+5	0.02	0.16	0.28	0.33	0.27	0.01
	2+4	-0.09	0.06	0.17	0.20	0.09	-0.26
$b_{1L}+b_{2L}$	3+3	-0.14	0.03	0.16	0.18	0.02	-0.39
	4+2	-0.13	0.10	0.28	0.33	0.15	-0.30
	5+1	0.03	0.35	0.57	0.62	0.48	0.05
	6+0	0.40	0.73	0.73	0.87	0.79	0.52

6 Conclusion

When studying competing rumors, information warfare or propaganda battles, the notion of agenda is of great importance. However, nearly none of related mathematical models takes into account the agenda-setting theory. The likely reason is that the most of these models are uncapable by their design to consider propaganda battles over more than one topic. This paper is aimed to start filling the pap. We consider a propaganda battle with two-component agenda to obtain a Blotto game which is quite natural in situations where competing (or belligerent) parties allocate their resources to several battlefields. In our case, the game is easily solved in pure strategies.

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