Geometric Modeling of Stress Visualization Based on the Functional-Voxel Method *

Sergey Pushkarev[1[0000-0002-4290-6455]] and Alexey Tolok[2[0000-0002-7257-9029]]

[1] Moscow State Technological University "STANKIN", Moscow, Russian Federation
sapushkarev@gmail.com
[2] Institute of Control Sciences V.A. Trapeznikov of Russian Academy of Sciences, Moscow, Russian Federation
tolok_61@mail.ru

Abstract. The visualization of the parameters of the stress state of a solid remains one of the parameters influencing the adoption of engineering decisions. For example, methods for determining finite elements (FEM), which make it possible to determine and visualize stress in the selected regions of the model. Applying element methods to analytically constructed models to localize the search for stress to its values at a point, however, will not lead to successful results. The paper discusses the principles of visualization of local stresses based on the functional-voxel method. The concept of a volume vector as a unit of volume distribution of a force vector in a solid isotropic medium is introduced. Geometrical foundations are proposed for computer representation of the stress unit in an isomorphic body based on a raster image. Geometric models of the stress tensor are constructed for the main site, the inclined platform. The principles of applying the functional-voxel model in the tasks of constructing complex objects are proposed. The application of the functional voxel method for discrete modeling of the deformation of a geometric object is illustrated by the example of a function that describes a rectangular plate.

Keywords: Discrete Geometric Model, Finite Element Method, Stress in a Solid, Functional Voxel Method, Volumetric Vector, Deformation Modeling.

1 Introduction

One of the key parameters that significantly affect engineering decisions is the parameters of the state of stress of a solid. However, if the issue of stress visualization in the selected grid regions, as it is realized, for example, in the finite element method is sufficiently illuminated and widely applied, then the problems of modeling and visualization of local stresses remain open. But the development of new approaches to modeling and visualization opens the possibility of solving these problems.

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Currently, researchers are working towards the development of scientific visualization and the introduction of analytically described geometric models into the design process. This direction is actively promoted by such directions as R-functional modeling (RFM), which got its start in the Laboratory of Applied Mathematics of IPMASH NAS of Ukraine under the guidance of academician of NAS of Ukraine V.L. Rvachev [1], as well as the functional-voxel modeling method, developed under the guidance of Professor A.V. Tolok in ICS RAS [2]. In the first case, the problems of the analytical description of the constructive approach to constructing a complex functional space by means of the mathematical apparatus are considered. This allows a single analytical representation to describe a geometric object of any complexity. The second method is aimed at constructing a voxel computer representation of a functional area of any dimension and complexity of description, leading to simplification of computer processing of such a model.

The study of the capabilities of the functional-voxel model for solving stress determination problems showed that it is designed to work with an analytical description of the problem statement and is not suitable for visualizing the results of calculations obtained by the traditional finite element method. This is due to the specifics of organizing the data of the functional-voxel model,
which differs from the organization of data from the surface models used in CAD.

The developed below tools for computer visualization of normal and tangential stresses using functional voxel models for use in engineering tasks lay the foundation for the further development of the functional voxel modeling method and interactive graphic modeling tasks for analytical CAD systems based on a voxel modular platform.

2 Volumetric vector

A volume vector should be understood as a geometric object defined by analogy with a conventional vector (a directed segment from the starting point having a direction angle γ and a value of ρ), only the direction function γ (γ) and the function of the value ρ (ρ) are
defined for the starting point. The volumetric vector is illustrated in Figure 1.

To construct the first function – function of the quantity $\rho(R_A)$ it is necessary to localize the point of force application by some unit neighborhood, i.e. sphere with a unit surface area $S^1 = 4\pi R^2$, where $R = 1/(2\sqrt{\pi})$.

Parameter R_A is the increment of the distribution radius of the force vector $S = 4\pi(R + R_A)^2$, thus:

$$S = 1 + 2\sqrt{\pi}\rho + 4\pi\rho^2 = 1 + \frac{R_A}{R} + 4\pi \qquad (1)$$

The increase in the area under the applied force acts inversely with the value, so the law can be written as $\rho(R_A) = 1/(1 + R_A/R + 4\pi {R_A}^2)$. In the case of the application of force to the surface of a solid body, the considered neighborhood of the point turns into a hemisphere, which means that the law changes to $\rho(R_A) = 2/(1 + R_A/R + 4\pi {R_A}^2)$ respectively.

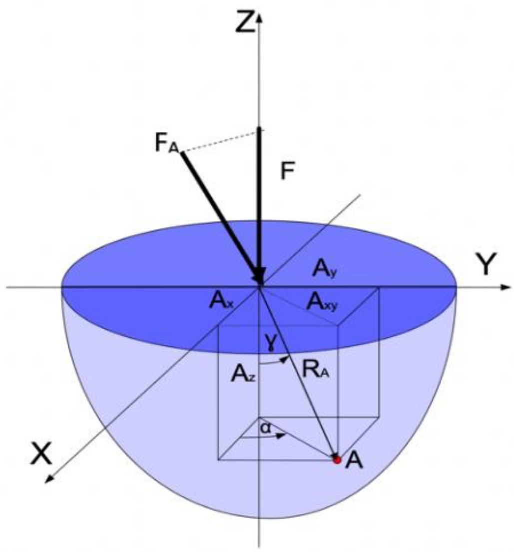


Fig.1. Volumetric vector.

Figure 2 shows the principle of projection of force F perpendicular to the main site of normal stress. The perpendicular to such a site is determined by the direction of the line passing between the point of the body under consideration A and the point of application of force (the initial exact volumetric vector). Power projection $F_A = Fcos\gamma$. The applied force must have the radius of the plane neighborhood of the application, the radius of the neighborhood is taken R.

Taking the body as an infinite bundle of bounded planes intersected at point A, we can imagine an infinite number of rotatable minimal neighborhoods with the unidirectional flow of force F applied to them.

Figure 3 demonstrates a separate case of such a turn of the neighborhood relative to the flow$F$, here there is a decrease in the number of flow elements (in the form of arrows) falling on the site of the neighborhood when turning through an angle γ. the rotation is indicated by an arrow. Given the obtained property, the projection F_A takes the following form: $F_A = F\cos\gamma\cos\gamma = Fcos^2\gamma$. Combining the functional laws $\rho(\rho)$ and $\gamma(\gamma)$ y means of multiplication, we obtain the general functional law of constructing the volumetric stress vector $\sigma = V(\rho(R_A), \gamma(\gamma))$:

$$\sigma = \frac{Fcos^2\gamma}{1 + 2\frac{R_A}{R} + 4\pi {R_A}^2}, \text{где } R = \frac{1}{2\sqrt{\pi}} \qquad (2)$$

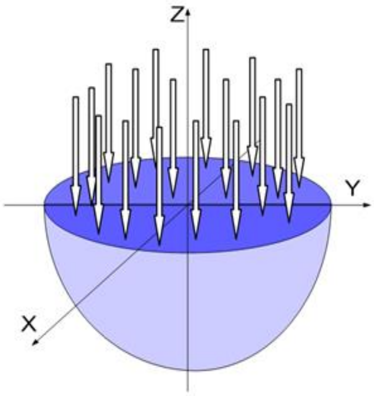


Fig.2. The distribution of the power flow on a single site near the point of application of force.

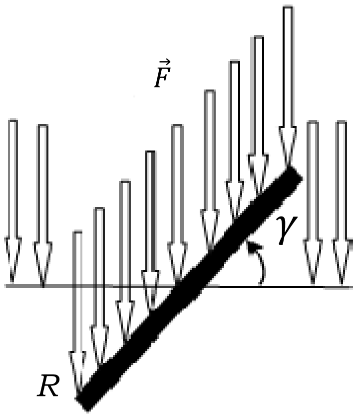


Fig.3. Change in load F when turning the platform.

In that case, if the origin of the coordinate system is set at the point of application of force, then $R_A(x_A, y_A, z_A) = \sqrt{x_A^2 + y_A^2 + z_A^2}$.

The resulting volumetric vector model is representable by the sum of two basic physical laws that determine the vector (direction, distance to the point of application), their geometric meaning is expressed by two laws. The first law can be represented by two states: the axial distribution of the volume vector and the radial distribution.

The axial distribution is shown in Figure 4 and is constructed by analogy with the Lambert law of light for a simple lighting model $I = I_l \cos\alpha$ where I − reflected light intensity, I_l − the intensity of the incident light and α − normal angle $\vec{n}$ к site reflection. For the case in question:

$$|\vec{F}_A| = |\vec{F}|\, cos\,\gamma = |\vec{F}|\, {}^{A_z}/_{R_A} \qquad (3)$$

The radial distribution does not depend on the angle of rotation of the reflection platform, preserving its value of the length of the applied vector (see Fig.5).

The law of temperature distribution and many wave processes can be attributed to the radial distribution.

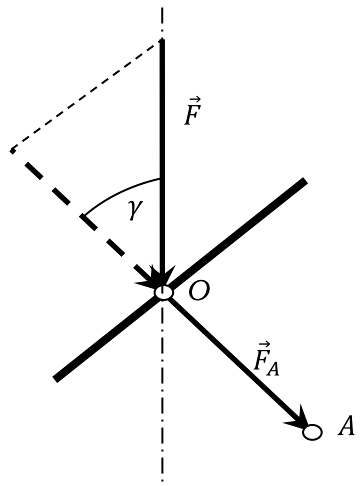


Fig.4. Lambert's cosine law. Axial distribution.

Raster representation of local geometric characteristics $|\vec{F}_A|$ axial law for a point P, selected in the body space relative to the point of application of force F is demonstrated by the intensity of the semitone in Figure 6.

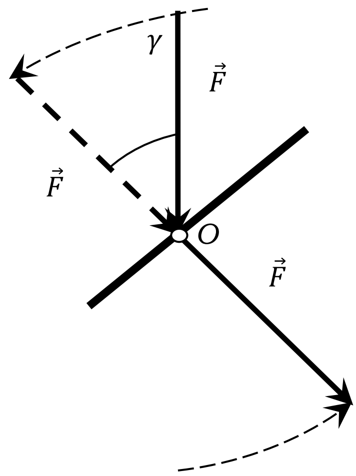


Fig.5. Radial distribution of the vector

The second law is related to the distance from the initial point of application of force:

$$1/_{S_A} = 1/_{(\pi R_A^2)} \quad (4)$$

Since the dependence on the area S_A, then the law is quadratic (see Fig.7), or rather hyperbolic. Both laws can be attributed to the geometric transformations of the object (by analogy: rotation, shift), which means that their product will give a general transformation that calculates the stress characteristic $|\vec{\sigma}|$ (see Fig.8).

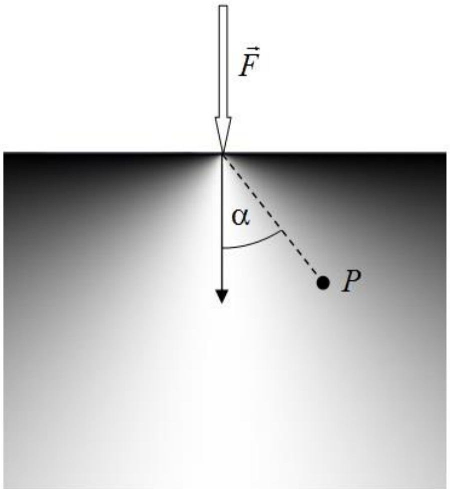


Fig.6. Display the axial distribution of the volume vector.

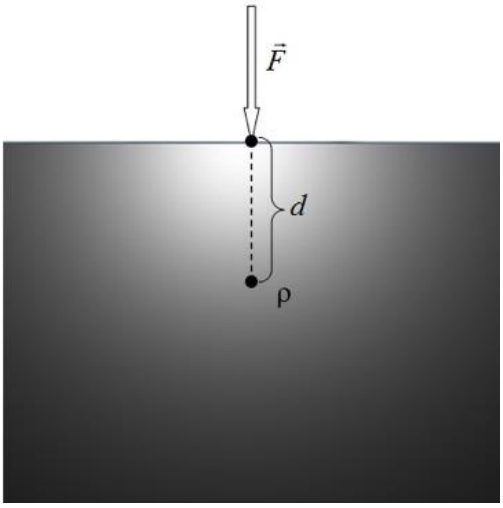


Fig. 7. Displays force versus distance from start point.

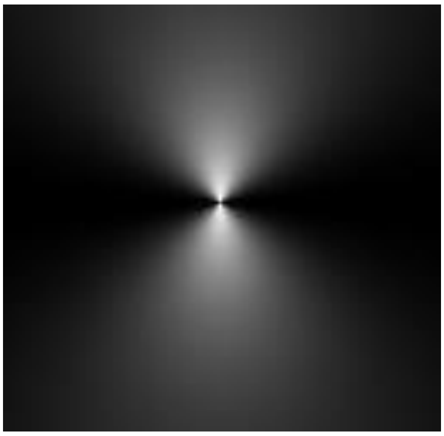


Fig. 8. The stress state of the loading unit of the simulated FVM

The volumetric vector allows you to build a geometric model of the stress tensor at point A for its main platform:

$$\sigma_{ij} = \begin{bmatrix} \frac{FA_zA_x}{\pi R_A^4} & 0 & 0 \\ 0 & \frac{FA_zA_y}{\pi R_A^4} & 0 \\ 0 & 0 & \frac{FA_z^2}{\pi R_A^4} \end{bmatrix} \quad (5)$$

And the geometric model of the stress tensor for an inclined platform:

$$\tau_{ij} = \begin{bmatrix} \sigma_{xx} & \frac{FA_{xy}A_x}{\pi R_A^4} & \frac{FA_{xy}A_z}{\pi R_A^4} \\ \frac{FA_{xy}A_x}{\pi R_A^4} & \sigma_{yy} & \frac{FA_{xy}A_y}{\pi R_A^4} \\ \frac{FA_{xy}A_z}{\pi R_A^4} & \frac{FA_{xy}A_y}{\pi R_A^4} & \sigma_{zz} \end{bmatrix} \quad (6)$$

For the correct visualization of the stress tensor at the point, the equilibrium at the point is also determined. For this, the law of paired tangential stresses is introduced into the geometric model.

Figure 9 shows the visualization of the local normal σ (a) distributed over the local area of application of force and the local tangential (b) stress $|\vec{\tau}|$ modeled in the RANOK 2D system.

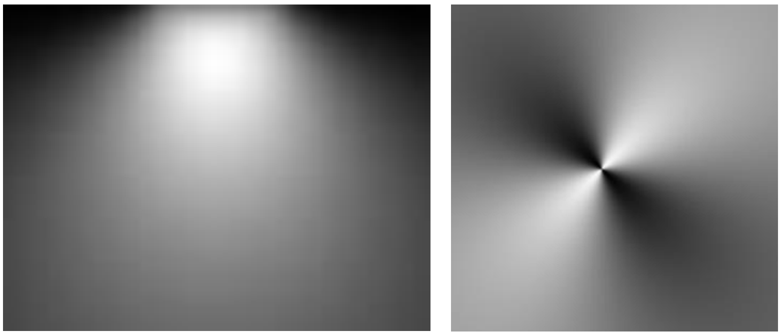


Fig.9. Visualization of the local normal σ (a) distributed over the local area of application of force and the local tangential (b) stress $|\vec{\tau}|$ modeled in the RANOK 2D system.

3 Discrete modeling of deformation of a geometric object

The following illustrates the discrete modeling of the deformation of a geometric object using the functional-voxel method is considered as the interaction of two functions - the description of a rectangular plate and the geometric form of loading, united by a common space and independent in its representation. This approach allows us to consider the transformation from the position of the form of loading, and from the position of the geometric object itself. For such a description, R-functional modeling is applicable, which allows one to analytically describe the space.

The formulation of the function space for describing the shape of the loading region ω is realized using the principle of the perceptual model [3], in which the body of the geometric object is filled with units and the surrounding space with zeros (see Fig.10).

In this way, a spatial object is formed where the unit area expresses the loading field, and the zero area excludes such a field, while maintaining the possibility of conversion.

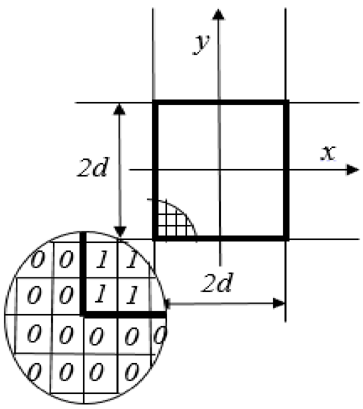


Fig.10. Perceptual square model

The example of the square function illustrates the process of forming a perceptual model in an R-functional way. The intersection of two bands of the same width *2d* (see Fig. 11), functionally described by $\omega_1 = d^2 - y^2$ and $\omega_2 = d^2 - x^2$ functionally describes the positive range of values of the square function with the negative region of the surrounding space. Moreover, each of these laws describes an infinitely distributed parabola along the chosen axis, intersecting the xOy plane at a distance d.

The R-functional intersection of such functions allows us to obtain a positive range of ω in the form of a square with sides 2d (see Fig.12):

$$\omega = \omega_1 + \omega_2 - \sqrt{\omega_1^2 + \omega_2^2}. \quad (7)$$

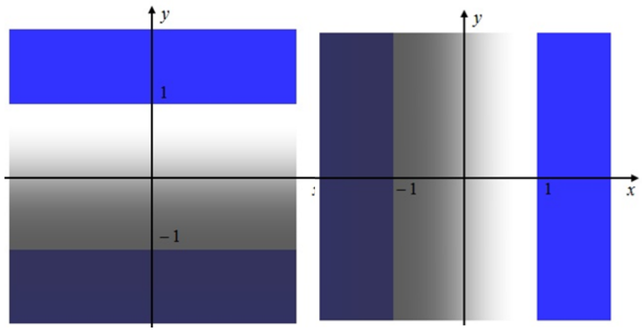


Fig.11. Display a positive function area ω_1 and ω_2

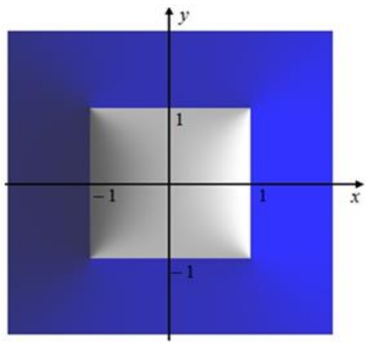


Fig. 12. Positive area of space ω «square»

The obtained range of values allows us to go on to describe the perceptual model directly, for which the space region of the function ω is reduced to the unit value of the positive region and zeroing of the negative region.

$$\omega_0^1 = \frac{\frac{\omega}{|\omega|} + 1}{2} \quad (8)$$

As a result of the described transformations, an M-image of the perceptual model can be obtained $C_{\omega_0^1} = \omega_0^1 P, (P = 255)$ (see Fig.13), here the positive area of the function is displayed in white ω, and black - negative.

The following is the process of modeling the function space $\omega_{пл}$, describing the geometrical object «plate».

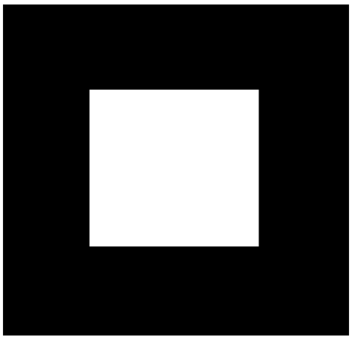


Fig. 13. Perceptual function image ω - «unit square»

«Plate» - rectangular prism specified by the parameters: $2a, 2b$ и $2c$ (see Fig.14) by analogy to the square described above. Here is the function space $\omega_1 = a^2 - x^2$ will have a positive range of values enclosed between parallel planes $x = a$ and $x = -a$. Function space $\omega_2 = b^2 - y^2$ will create a positive range of values between the planes $y = b$ and $y = -b$. A function space $\omega_3 = c^2 - z^2$ will take positive values between the planes given by the equations z$= c$ and z$= -c$.

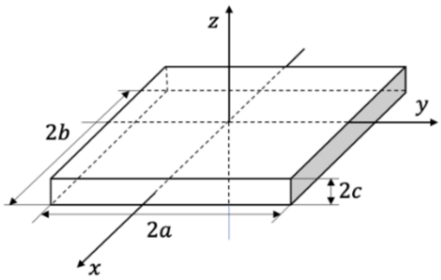


Fig. 14. Drawing plate to describe the function space.

Thus, to describe the positive range of values of the function space, concluded between all pairs of planes, describing the space of a given plate with a size of $2a \times 2b \times 2c$, we can use the R-functional modeling apparatus:

$$\omega_{12} = \omega_1 + \omega_2 - \sqrt{\omega_1^2 + \omega_2^2} \quad (9)$$

$$\omega_{pl} = \omega_{12} + \omega_3 - \sqrt{\omega_{12}^2 + \omega_3^2} \quad (10)$$

Figure 15 shows the positive region of the space of the plate function $\omega_{пл}$, obtained in the RANOK 3D system.

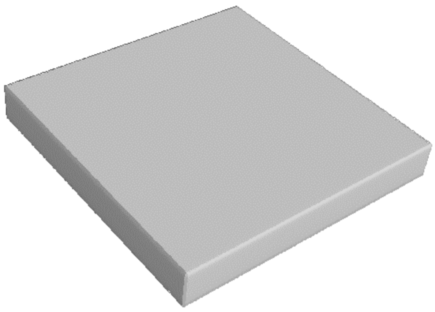
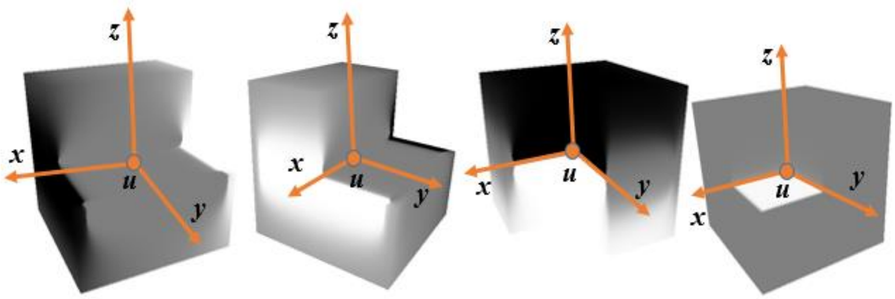


Fig. 15. Drawing plate to describe the function space.

This model is the initial one for the further construction of the algorithm for calculating the geometric transformation based on the application of the field of volume vectors given on the domain ω_0^1. It (FV-model) describes a discrete representation of a given space of the function ω_{pl} in the form of a set of five voxel M-images in the form of a set of five voxel M-images that graphically display information about the components n_1, n_2, n_3, n_4, n_5 (see Fig.16).



$C_1 \to n_1$ $C_2 \to n_2$ $C_3 \to n_3$ $C_4 \to n_4$

Fig. 16. Voxel M-images making up the FV-model of the function space

The transition to a discrete model allows you to develop a computer algorithm for geometric transformation of function space. According to the principles of functional voxel modeling, for computer calculations, the local function u_{pl} at the point in question will be used

$$u_{pl} = \frac{n_5}{n_4} - \frac{n_1}{n_4}x - \frac{n_2}{n_4}y - \frac{n_3}{n_4}z \tag{11}$$

The following is an algorithm for converting function space points ω_{pl} relative to a given field of volume vectors specified by the model ω_0^1. The essence of the algorithm is the calculation of stress values $(\sigma_x, \sigma_y, \sigma_z)$ and (τ_x, τ_y, τ_z), created by a given field of volume vectors through a perceptual model ω_0^1, at each point of a given space of a function with coordinates (x, y, z). The obtained values will determine the spatial shift along the coordinate axes to determine the new function value ω_{pl} for the current point in question. Thus, function ω_{pl} changes its values on a given space and, thereby, affects the shape of the positive region of its values. The given region of volume vectors is continuous. The discrete model of the M-image (see. Fig 13) allows you to discretely distribute the points of application of volume vectors with uniform filling density of a single space. Using the basic calculation formulas at the point of the stress value based on the volume vector, it is possible to determine the stress field in the region described by the units of the function ω_0^1, which is the sum of unit stresses:

$$(\omega_0^1) = \sum_{x=-40}^{40} \sum_{y=-40}^{40} \left(\frac{F cos^2\gamma}{1 + 2\frac{R_A}{r} + 4\pi {R_A}^2} \right) \omega_0^1(x, y) \tag{12}$$

$$\tau(\omega_0^1) = \sum_{x=-40}^{40} \sum_{y=40}^{40} \left(\frac{F_A \cdot sin2\gamma}{2(1 + 2\frac{R_A}{r} + 4\pi {R_A}^2)} \right) \omega_0^1(x, y) \tag{13}$$

where $R_A = \sqrt{x^2 + y^2 + z^2},\ r = \frac{1}{2\sqrt{\pi}},\quad cos\gamma = \frac{z}{R_A},\ sin\alpha = \frac{x}{\sqrt{x^2+y^2}}$.

Parameters of spherical coordinates of a volume vector (γ, α) allow you to decompose into components each of the stresses of the above amounts.

$$\begin{gathered}
\sigma_x = \sigma(\omega_0^1)\ sin\,\gamma cos\alpha; \\
\sigma_y = \sigma(\omega_0^1)\ sin\,\gamma\ sin\,\alpha\,; \\
\sigma_z = \sigma(\omega_0^1) cos\,\gamma\,; \\
\tau_x = \tau\ (\omega_0^1)\ cos\,\gamma\ cos\,\alpha\,; \\
\tau_y = \tau\ (\omega_0^1)\ cos\,\gamma\ sin\,\alpha\,; \\
\tau_z = \tau(\omega_0^1)\ \ sin\,\gamma.
\end{gathered} \tag{14}$$

Relative spatial shift along each axis, taking into account the obtained projections of local stresses $(\sigma_x, \sigma_y, \sigma_z)$ и (τ_x, τ_y, τ_z) calculated as:

$$\Delta x = \sigma_x + \tau_x, \quad \Delta y = \sigma_y + \tau_y, \quad \Delta z = \sigma_z + \tau_z \tag{15}$$

The essence of the transformation is that the coordinates of each point in the function space ω_{pl} submitted to the calculation taking into account the received bias $(x + \Delta x, y + \Delta y, z + \Delta z)$, but retain their spatial position (x, y, z), which leads to a relative change in the values of the function ω'_{pl}, while maintaining the continuity and differentiability of the transformed space region (see Fig.17).

Fig. 17. Transformed function image ω'_{pl}.

This conversion T^{σ} belongs to the class of spatial, and the resulting space of the function after applying such a transformation retains its smoothness and continuity. The presented geometric models do not consider some physical parameters that would be used in the physical formulation of the described problems. For example, Young's modulus characterizes the physical properties of the material and is certainly necessary in the case of physical calculation, but it does not influence the geometric model of deformation. Figure 18 shows examples of M-images for a different description of the function ω_0^1 and Figure 19 shows the result of the conversion for each of these images.

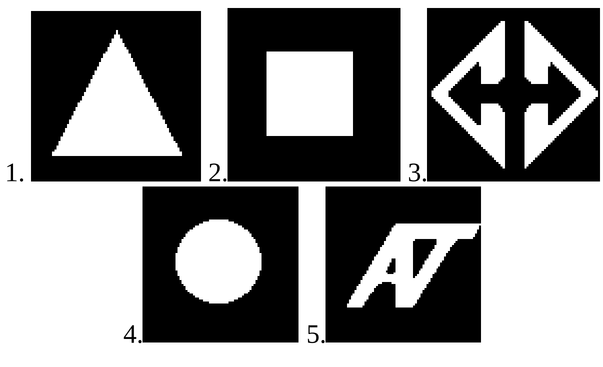


Fig. 18. M-images of various forms of function space ω_0^1.

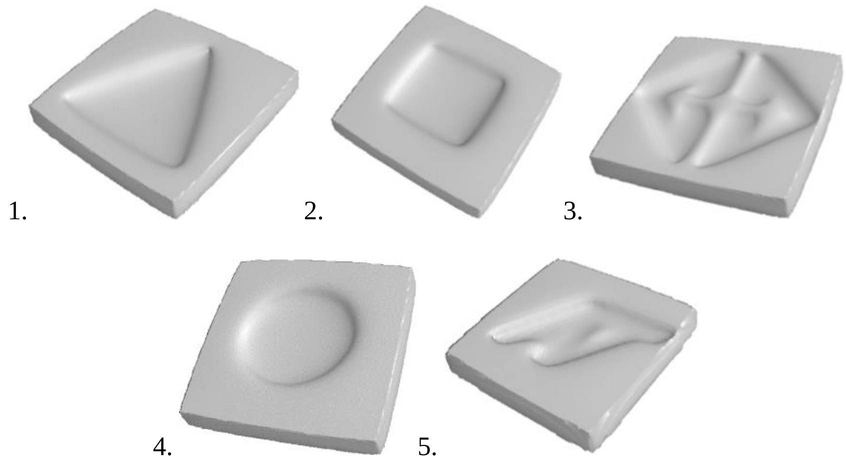


Fig.19. Spatial transform results

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