Visualization of the Process of Static Buckling of a Micropolar Meshed Cylindrical Panel^{*}

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Abstract. Process visualization of static stability loss in mechanics is shown by the micropolar meshed cylindrical panel example with two families of mutually perpendicular ribs. The mathematical model of the panel's behavior is based on the Kirchhoff-Love hypotheses. The micropolar theory is applied to account for scale effects. Geometric nonlinearity is taken into account according to the theory of Theodor von Karman. The mesh structure is taken into account based on the Pshenichnov I. G. continuum model. Visualization of numerical results using Autodesk 3ds Max software made it possible to more clearly assess the phenomenon of static buckling of the shell in question. Visualization of the results using 3D made it possible to establish that an increase in the distance between the edges of the mesh panel and an increase in the parameter depending on the size does not change the bending shape of the panel, as well as the diagrams of moments and forces at subcritical and supercritical loads.

Keywords: Meshed Panel, Micropolar Theory, Buckling, Geometric Nonlinearity.

1 Introduction

Subsequent paragraphs, however, are indented Meshed structural elements are widely used in engineering practice. The development of nano-technologies leads to supplement of studying the behavior of meshed elements at the micro- and nano-scale level. [1-4]. The nowaday question is qualitative visualization of the results of numerical experiments [5-6]. The presentation of the results of numerical experiments not in tabular form, but in the form of 2D and 3D graphs will allow the deeper understanding

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of the behavior of elements of mechanical structures under the influence of various kinds of factors. The description of the meshed structure of the structural elements is mainly based on two design models: continuous [7-8] and discrete [9-12]. Such theories as the micropolar moment theory of elasticity [13-16], the nonlocal theory of elasticity [17-19], the gradient theory of elasticity [20] and surface elasticity [21] are being developed today for to simulate scale effects in the continuum.

Today, there is a large number of studies of the full-sized statics and dynamics of the meshed structures [7, 8, 11, 22]. However, there are very few works devoted to the study of the behavior of the meshed plates and shells based on theories which are built on the effects of scale. [23-26].

2 Problem statement

The object of study is a shallow cylindrical panel rectangular in plan, which occupies an area $\Omega = \left\{-c \le x \le c; -b \le y \le b; -\frac{h}{2} \le z \le \frac{h}{2}\right\}$ in space R^3 . The nonzero components of the strain tensor in the case of Kirchhoff – Love hypotheses and T. von Karman's theory can be written as:

$$e_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 - z \frac{\partial^2 w}{\partial x^2}; \ e_{xy} = \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right) - z \frac{\partial^2 w}{\partial x \partial y},$$

$$e_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 - k_y w - z \frac{\partial^2 w}{\partial y^2};$$
(1)

here u,v,w are axial displacements of the middle surface of the plate in the directions x, y, z respectively, k_y is geometric parameter of curvature. The panel material is considered as Cosserat pseudo-continuum with cramped rotation of the particles. Along with the stress field, the moment stresses are also taken into account. It is assumed that the fields of displacements and rotations are not independent. The components of the symmetric bending-torsion tensor, taking into account of the accepted hypotheses and assumptions, can be written as follows:

$$\chi_{xx} = \frac{\partial^2 w}{\partial x \partial y}; \ \chi_{yy} = -\frac{\partial^2 w}{\partial y \partial x}; \ \chi_{xy} = \frac{1}{2} \left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2} \right); \ \chi_{xz} = \frac{1}{4} \left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} \right);$$

$$\chi_{yz} = \frac{1}{4} \left(\frac{\partial^2 v}{\partial y \partial x} - \frac{\partial^2 u}{\partial y^2} \right).$$
(2)

We take the defining relations for the panel material in the form:

$$\sigma_{xx} = \frac{E}{1-\nu^2} \left[e_{xx} + \nu e_{yy} \right], x \rightleftharpoons y, \ \sigma_{xy} = \frac{E}{(1+\nu)} e_{xy}, \tag{3}$$

where σ_{ij} are the components of the stress tensor, m_{ij} are components of the moment tensor of higher order, *E* is Young's modulus, ν is Poisson's ratio, γ is additional independent length parameter.

The equations of the motion of an element of a smooth panel, equivalent to a meshed panel, the boundary and initial conditions are obtained from the Ostrogradsky – Hamilton's energy principle . subject of the study is the meshed panel under the influence of normal distributed load. The panel consists of n sets, densely spaced ribs of the same material. According to the continuum G. I. Pshenichnov's model the regular rib system can be replaced with a continuous layer. The stresses arising in the equivalent smooth panel connected with the stresses in the ribs which make up the angles φ_j with the abscissa axis will have the form:

$$\sigma_{xx} = \sum_{j=1}^{n} \frac{\sigma_x^j \delta_j \cos^2 \varphi_j}{a_j}, \ \sigma_{yy} = \sum_{j=1}^{n} \frac{\sigma_x^j \delta_j \sin^2 \varphi_j}{a_j}, \ \sigma_{xy} = \sum_{j=1}^{n} \frac{\sigma_x^j \delta_j \cos \varphi_j \sin \varphi_j}{a_j},$$
$$m_{xx} = \sum_{j=1}^{n} \frac{m_x^j \delta_j \cos^2 \varphi_j}{a_j}, \ m_{yy} = \sum_{j=1}^{n} \frac{m_x^j \delta_j \sin^2 \varphi_j}{a_j}, \ m_{xy} = \sum_{j=1}^{n} \frac{m_x^j \delta_j \cos \varphi_j \sin \varphi_j}{a_j},$$
$$m_{xz} = \sum_{j=1}^{n} \frac{m_{zx}^j \delta_j \cos \varphi_j}{a_j}, \ m_{yz} = \sum_{j=1}^{n} \frac{m_{zx}^j \delta_j \sin \varphi_j}{a_j},$$
$$m_{yz} = \sum_{j=1}^{n} \frac{m_{zx}^j \delta_j \cos \varphi_j}{a_j}, \ m_{yz} = \sum_{j=1}^{n} \frac{m_{zx}^j \delta_j \sin \varphi_j}{a_j}$$

Where a_j is distance between edges of j-th sets, δ_j is the thickness of the ribs, voltage index j are rods. Stresses with index j refer to ribs. The physical relations for the meshed plate are determined based on the Lagrange multiplier method (5).

$$\sigma_x^j = \sigma_{xx} \cos^2 \varphi_j + \sigma_{yy} \sin^2 \varphi_j + \sigma_{xy} \cos \varphi_j \sin \varphi_j; \ \tau^j = \sigma_{xz} \cos \varphi_j + \sigma_{yz} \sin \varphi_j; m_x^j = m_{xx} \cos^2 \varphi_j + m_{yy} \sin^2 \varphi_j + m_{xy} \cos \varphi_j \sin \varphi_j;$$
(5)

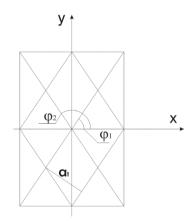


Fig.1. Panel mesh geometry.

The obtained physical relations (5) and expressions which relate the stresses arising in the equivalent smooth panel with the stresses in the ribs (4) will allow us to write the relations for the forces and moments of the smooth panel of the equivalent meshed panel. Substituting the got relations for the forces and moments into the equations of the motion of the smooth panel, we obtain the equations of the motion of the meshed micropolar panel. Later, we will consider the panel with two sets of ribs: $\varphi_1 = 45^\circ, \varphi_2 = 135^\circ, \ \delta_1 = \delta_2 = \delta, \ a_1 = a_2 = a$ (Fig.1).

$$\begin{split} & 2(\nu-1)\frac{\partial^{2}u}{\partial y^{2}} - 2(3+\nu)\frac{\partial^{2}v}{\partial x\partial y} - 4(\nu+1)\frac{b^{2}}{c^{2}}\frac{\partial^{2}u}{\partial x^{2}} + 4k_{y}(\nu+1)\frac{\partial w}{\partial x} + 2(\nu-1)\frac{\partial^{2}w}{\partial y^{2}}\frac{\partial w}{\partial x} - \\ & -2(3+\nu)\frac{\partial w}{\partial y}\frac{\partial^{2}w}{\partial x\partial y} - 4(1+\nu)\frac{b^{2}}{c^{2}}\frac{\partial w}{\partial x}\frac{\partial^{2}w}{\partial x^{2}} + \gamma^{2}(\nu-1)\left(-\frac{h^{2}}{b^{2}}\frac{\partial^{4}u}{\partial y} + \frac{h^{2}}{b^{2}}\frac{\partial^{4}v}{\partial x\partial y^{3}} - \\ & -\frac{h^{2}}{c^{2}}\frac{\partial^{4}u}{\partial x^{2}\partial y^{2}} + \frac{h^{2}}{c^{2}}\frac{\partial^{4}v}{\partial x^{3}}\right) = \frac{8a(\nu^{2}-1)}{\delta}\left(\frac{\partial^{2}u}{\partial t^{2}} + \varepsilon\frac{\partial u}{\partial t}\right) \\ & -4(1+\nu)\frac{\partial^{2}v}{\partial y^{2}} - 2(3+\nu)\frac{b^{2}}{c^{2}}\frac{\partial^{2}u}{\partial x\partial y} + 2(\nu-1)\frac{b^{2}}{c^{2}}\frac{\partial^{2}v}{\partial x^{2}} + 4k_{y}(1+\nu)\frac{\partial w}{\partial y} - 4(1+\nu)\frac{\partial w}{\partial y}\frac{\partial^{2}w}{\partial y^{2}} - \\ & -2(3+\nu)\frac{b^{2}}{c^{2}}\frac{\partial w}{\partial x}\frac{\partial^{2}w}{\partial y} + 2(\nu-1)\frac{b^{2}}{c^{2}}\frac{\partial^{2}w}{\partial y}\frac{\partial w^{2}}{\partial x^{2}} + \gamma^{2}(\nu-1)\left(\frac{h^{2}}{c^{2}}\frac{\partial^{4}u}{\partial x\partial y^{3}} - \frac{h^{2}}{c^{2}}\frac{\partial^{4}v}{\partial y}\frac{\partial y}{\partial y^{2}} - \\ & -2(3+\nu)\frac{b^{2}}{c^{2}}\frac{\partial w}{\partial y}\frac{\partial^{2}w}{\partial x} + 2(\nu-1)\frac{b^{2}}{c^{2}}\frac{\partial w}{\partial y}\frac{\partial w^{2}}{\partial x^{2}} + \gamma^{2}(\nu-1)\left(\frac{h^{2}}{c^{2}}\frac{\partial^{4}u}{\partial x\partial y^{3}} - \frac{h^{2}}{c^{2}}\frac{\partial^{4}v}{\partial y}\frac{\partial y}{\partial y^{2}} - \\ & -2(3+\nu)\frac{b^{2}}{c^{2}}\frac{\partial w}{\partial y}\frac{\partial w}{\partial x} + 2(\nu-1)\frac{b^{2}}{c^{2}}\frac{\partial w}{\partial y}\frac{\partial w^{2}}{\partial x^{2}} + \gamma^{2}(\nu-1)\left(\frac{h^{2}}{c^{2}}\frac{\partial^{4}u}{\partial x\partial y^{3}} - \frac{h(1+\nu)}{c^{2}}\frac{\partial w}{\partial y}\frac{\partial w}{\partial y^{2}} - \\ & -2(3+\nu)\frac{b^{2}}{c^{2}}\frac{\partial w}{\partial y}\frac{\partial w}{\partial x} + 2(\nu-1)\frac{b^{2}}{c^{2}}\frac{\partial w}{\partial y}\frac{\partial w^{2}}{\partial x^{2}} + \gamma^{2}(\nu-1)\left(\frac{h^{2}}{c^{2}}\frac{\partial^{4}u}{\partial x\partial y^{3}} - \frac{h(1+\nu)}{c^{2}}\frac{\partial w}{\partial y}\frac{\partial w}{\partial y^{2}} + 2(\nu-1)\frac{b^{2}}{c^{2}}\frac{\partial w}{\partial y}\frac{\partial w}{\partial y} + 2(\nu-1)\frac{b^{2}}{c^{2}}\frac{\partial^{4}w}{\partial x^{2}} + \gamma^{2}(\nu-1)\left(\frac{b^{2}}{c^{2}}\frac{\partial^{4}w}{\partial x^{2}} + \frac{h^{2}}{c^{2}}\frac{\partial^{4}w}{\partial x^{2}} + \frac{h^{2}}{c^{2}}\frac{\partial^{4}w}{\partial y} + \frac{h^{2}}{c^{2}}\frac{\partial^{4}w}{\partial x^{2}} + \frac{h$$

Initial and boundary conditions should be added to the equations.

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In the experiments were taken the zero initial conditions and the fixed boundary conditions:

$$u = v = w = 0, \frac{\partial u}{\partial x} = 0, \frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial x} = 0, \frac{\partial v}{\partial y} = 0, \frac{\partial w}{\partial x} = 0, \frac{\partial w}{\partial y} = 0 \quad npu \quad x = \pm 1, y = \pm 1.$$
(7)

Static problems in the theory of plates and shells have traditionally been solved using various approximate methods. They allow to modify the system of partial differential equations to the system of nonlinear algebraic equations, which is further linearized. In this article, the solution of static problems will be presented using the establishment method, which was for shells by I.V. Feodosiev first applied. In the method of establishing setting, the solving of the system of partial differential equations reduces to solving the Cauchy problem for the system of ordinary differential equations, which is initially linear in time. This approach has some advantages. The establishment method has high accuracy, as it can be related to iterative methods. Here, each time step is new approximation to the exact solution of the problem. In addition, the establishment method isn't very sensitive to the initial choice of the approximation. Solving the Cauchy problem for $\varepsilon = \varepsilon_{sp}$, for the number of values of the normal time constant w_i . parameter of the load q_i , we obtain the sequence of deflections Based on these data, the relationship w(q) is constructed and the stress-strain state of the system is studied. It should be noted that it is necessary to pay serious attention to the choice ε_{sp} .

In some cases, if energy dissipation is not taken into account, the critical dynamic value of the load can be approximately half the static critical value of the load. In this article, for to solve the static problem, the system of partial differential equations was reduced in spatial coordinates by the finite difference method with a second-order approximation to the Cauchy problem. The Cauchy problem was solved by Newmark's method. It was experimentally chosen $\varepsilon_{sp} = 10$. This value gives the smaller number of

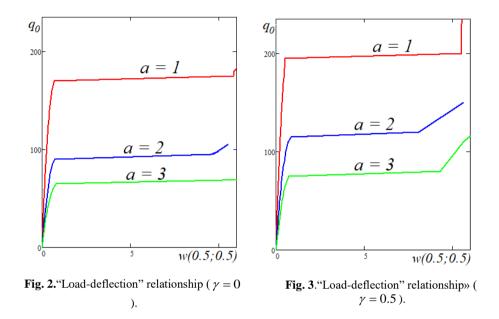
iterations solving the problem by the establishment method.

When solving problems in mechanics numerically, the results are presented in the form of numerical tables. The analysis of the results in this form causes great difficulties. For the qualitative assessment of the information contained in the tables, its high-quality graphic visualization is necessary. In this work, the software used Autodesk 3ds Max. Compared to the many existing programs developed for visualization of numerical solutions, Autodesk 3ds Max has a large set of utilities that allow you not only to get good image resolution, but also to get creative with the visualization itself. Apply various effects to the image: multi-colored lighting from different angles, add a specular reflection to the image.

3 Numerical experiment

Consider a mesh cylindrical panel under the action of a transverse uniformly distributed static load. Fig. 2-3 shows of the dependence "static load-deflection".

The studies were carried out depending on the increase in the distance between the



edges a = 1, 2, 3 for a mesh panel with curvature parameters $k_y = 48$. As the distance between the panel edges increases, the structure becomes softer, i.e. the bearing panel Only two levels of headings should be numbered. Lower level headings remain unnumbered; they are formatted as run-in headings.

becomes softer, i.e. the bearing panel capacity decreasing. The increase in the distance between the panel edges did not affect the panel bending form, as well as the diagrams of moments and forces under subcritical and supercritical loads. An increase in the size-dependent parameter increases the bearing capacity of the panel.

Process visualization of the static stability loss was carried out using the Autodesk 3ds Max program. The table 1-2 shows the diagrams of deflection, average moment $\left(M_{avg} = \frac{M_x + M_y}{2}\right)$ and average forces $\left(N_{avg} = \frac{N_x + N_y}{2}\right)$ of the panel at a load of $q_0 = 40$ (subcritical load) and after "clap" at $q_0 = 120$ (supercritical load), at a=3 and

 $q_0 = 40$ (supercritical load) and after clap at $q_0 = 120$ (supercritical load), at a=5 and a size-dependent parameter $\gamma = 0$; 0.5.

At $q_0 = 40$, the deflection plot has a dome-shaped shape, after "clap" at $q_0 = 120$, the shape is also dome-shaped, but the apex is sharper.

The visualization of the moment shows that before the "clap", the moment values in the center of the shell are positive, while the moment values are negative at the edges of the shell. The moment diagram changes shape after the "clap". The visualization of the results the results the "clap". The visualization of the results shows that after the "clap", the moment has a maximum value in the center of the shell. Visualization of the Process of Static Buckling of a Micropolar Meshed Cylindrical Panel 7

Static buckling leads to a change in the shape of the force diagram. After the "clap" forces in the quarters change sign. The graphs show that with the value of the size-dependent parameter, changes in the distribution of forces in the shell are more pronounced.

Table 1. Diagrams of deflections, moments and forces for the shell $\gamma = 0$.

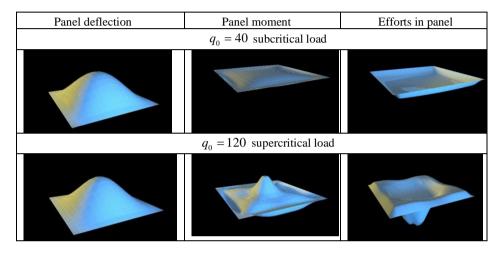
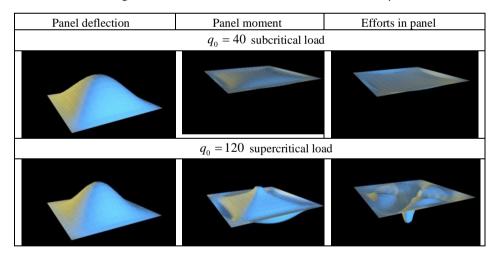


Table 2. Diagrams of deflections, moments and forces for the shell $\gamma = 0.5$



4 Conclusion

The article presents a visualization of the process of loss of static stability of a cylindrical panel with a micropolar mesh. Static buckling is accompanied by a change in the diagrams of moments and forces at the subcritical and supercritical points. Changes

in the force diagrams are more noticeable when moment stresses are taken into account. Visualization of the results using 3B made it possible to establish that an increase in the distance between the edges of the mesh panel and an increase in the parameter depending on the size does not change the bending shape of the panel, as well as the diagrams of moments and forces at subcritical and supercritical loads.

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