Deductive vs. Inductive Instructions: Evaluating the Predictive Powers of Cognitive Models for Conditional Reasoning

Sara Todorovikj[1,2] and Marco Ragni[1,2]

[1] Cognitive Computation Lab, University of Freiburg, 79110 Freiburg, Germany
[2] Danish Institute for Advanced Study, University of Southern Denmark
{todorovs,ragni}@informatik.uni-freiburg.de

Abstract. Throughout the years, the question how humans reason with conditionals has been extensively researched by various disciplines due to its importance not only in science, but also our everyday life. A vast amount of cognitive models have been developed in order to get a better insight into how individuals interpret and reason with 'If'. Todorovikj and Ragni [21] proposed a predictive modeling testing paradigm for probabilistic conditional reasoning models. Instead of evaluating models based on their ability to fit aggregate data, the focus was shifted to the individual by challenging cognitive models to predict a participant's answer on a scale from 0 to 100. In this work, we continue the challenge of predicting probabilistic endorsements by taking data from Singmann and Klauer [19] who examine the influence of *deductive vs. inductive* instructions. We evaluate an established probabilistic cognitive model by Oaksford and Chater [11] and a proposed probabilistic approach based on mental models [21, 20]. Moreover, we take Oberauer's formalizations [12] of the Mental Model Theory [5] and the Suppositional Theory [4] and adapt them to represent probabilistic data in this new predictive testing paradigm. The models' parameter values are evaluated to examine their ability to represent the influence of instruction and content effects.

Keywords: Conditional Reasoning · Predictive Modeling · Individuals · Instruction Effect · Content Effect

1 Introduction

On a daily basis, we gather knowledge and information about ourselves and our surroundings that we use to infer conclusions by processes of *reasoning*. For many years, scientists work towards understanding *how* we reach these conclusions and what *cognitive processes* lead to them. In the 60s, a *deductive* path has been followed, assuming that logic is the basis for reasoning [4]. However, it has been repeatedly shown that humans are not always logical beings, the content of a premise often makes us question its certainty in a way that standard deductive logic cannot grasp [19, 8]. Acknowledging these uncertainties, interpreting them as *probabilities* and switching the focus on the *inductive* strength of arguments creates a major shift leading to a new *Bayesian paradigm* [13]. Proponents of both paradigms have developed a number of *cognitive models* and experiments that represent and examine reasoning mechanisms corresponding to the paradigm's theories and assumptions. Singmann and Klauer [19] focus on the *conditional* reasoning domain. Through two experiments they provide evidence for a dissociation of the two reasoning modes: *deductive* and *inductive* and show the need for theories to explicitly account for the effect of instruction.

Conditionals are statements of the form "If X then Y" (also written $X \rightarrow Y$, where X is the *antecedent* and Y the *consequent*) that often describe a causal relationship between two propositions X and Y. Given a conditional (*major premise*) and a state of a proposition (*minor premise*), a *conclusion* can be *inferred* about the state of the other proposition. We consider the four major inference forms *modus ponens* (MP), *modus tollens* (MT), *affirming the consequent* (AC) and *denying the antecedent* (DA), as shown in Table 1.

Table 1: Inference Forms

Premise	MP	AC	DA	MT
Major	$X \rightarrow Y$	$X \rightarrow Y$	$X \rightarrow Y$	$X \rightarrow Y$
Minor	X	Y	$\neg X$	$\neg Y$
Conclusion	Y	X	$\neg Y$	$\neg X$

In causal conditionals the *necessity* and *sufficiency* of X for Y can be questioned and manipulated through two types of counterexamples: *alternatives* and *disablers*. Alternatives enable Y to occur even if X hasn't and decrease the perceived necessity. On the other hand, disablers prevent Y from being true even if X is, decreasing the perceived sufficiency. Conditionals whose content has many alternatives but few disablers associated with it are called *prological*, ones with few alternatives but many disablers are *counterlogical*, and if a conditional has both many alternatives and disablers then it is called *neutral* [19].

In previous work [21], we evaluated conditional models in a new *predictive* evaluation setup, motivated by Riesterer et al.'s approach for syllogistic reasoning [16]. We showed that it is possible for models to predict a *probabilistic endorsement* of a conclusion in the range 0-100, which is more challenging than a dichotomous 'yes' or 'no' answer. The focus was to predict an *individual's* answer, increasing the difficulty due to the varying *subjective* influence of disablers and alternatives. Additionally we proposed a probabilistic approach based on mental models, ϵ-MMT, which showed to be a valuable competitor among other Bayesian models.

Our main goal is to once again challenge the predictive capabilities of cognitive models, this time on Singman and Klauer's data sets [19]. We take formalizations [12] of the Mental Model Theory [5, 18] and Suppositional Theory [4], adapt them to represent *probabilistic* data and compare them to Oaksford and Chater's Bayesian model [11, 9] and ϵ-MMT [21, 20]. Moreover, we will analyze whether the models' parameter values can account for the instruction and content effects.

2 Data and Cognitive Models

We consider experimental data presented and analyzed by Singmann and Klauer [19]. There are two experiments, Exp. 1 and Exp. 2, aiming to establish a double dissociation between *deductive* and *inductive instructions* when validity and plausibility of the conditional problems are pitted against each other.

Under deductive instructions participants were asked to assume the truth of the premises and disregard background knowledge when judging the conclusion's validity. Under inductive instructions, they were encouraged to use background knowledge when judging the conclusion's probability. In both experiments participants were provided with prological and counterlogical conditionals. Additionally, in Exp. 2 they were also given neutral conditionals. 40 participants took part in Exp. 1, 20 received deductive instructions and 20 inductive. In Exp. 2, 27 participants received deductive instructions and 28 inductive, making it a total of 55 participants. In both experiments, they provided endorsements in the range 0-100 for the four inference forms for conditionals with *everyday* contents. Table 2 shows content examples under deductive and inductive instructions.

Table 2: Content example from Singman and Klauer's experiments [19].

Instructions	Content
Deductive	If a campfire goes out, then water has been poured on it. A campfire goes out. How *valid* is the conclusion that water has been poured on it?
Inductive	If a campfire goes out, then water has been poured on it. A campfire goes out. How *likely* is it that water has been poured on it?

2.1 Mental Model Theory (MMT)

The Mental Model Theory (MMT) assumes that when individuals are presented with some information, they build a mental representation of it using *mental models*. They aim to reach a conclusion based on the maintained information, and often, individuals would engage in a search for counterexamples to the conclusion. If their search is successful, they would no longer accept the conclusion [7]. A *mental model* consists of the truth states of the propositions in the premise. Given a conditional premise "If X then Y", the initial mental model that an individual would construct is the one where both propositions are true, X Y.

The MMT assumes that once the initial model is created it triggers the recollection of relevant facts and knowledge [5]. Those facts can either serve as evidence that the initial model is correct or will stimulate a search for alternatives and lead to extending the mental model representation in a second process.

The extended mental model representation is also called a *fleshed-out representation*, as shown in Table 3. It contains models that describe cases where X is false (written as ¬X). The fleshed-out representation consists of all the possible combinations of truth-values for the propositions X and Y for which the conditional "If X then Y" is true. Johnson-Laird and Byrne call representing only what is true and not false *the principle of truth* [6]. This coincides with the material implication definition which is the leading interpretation of conditionals in the deductive paradigm [2].

Table 3: Representation of a conditional premise "If X then Y" with mental models

Premise	Mental Model	Fleshed-out Models
If X then Y	X Y	X Y
	...	¬X ¬Y
		¬X Y

Later on we present and use a formalization of the MMT that is based on equations provided by Schroyens et al. [18] who revise the theory as presented by Johnson-Laird and Byrne [5] and present an alternative model within the mental models approach, focusing on the stage where individuals validate the conclusion based on their mental model representation. They assume that when looking for counterexamples individuals do not construct all possible alternative models but rather aim for the model that would falsify the inference. Based on the original model's theory, when looking for counterexamples individuals only test whether alternative models are consistent with the premises. Schroyens et al. [18] on the other hand, take background knowledge about the content into consideration and suppose that counterexamples are retrieved from long-term memory, which is where this theory stops being a theory of deductive reasoning [12]. With this modification the material implication interpretation of conditionals within the theory is abandoned because now the X ¬Y model can actually be retrieved from long-term memory and become a part of the mental model representation. That leads to the possibility of refuting the logically valid MP and MT inferences, which is a consequence of the presence of disablers as described by the Suppression effect [1].

Evans [3] has criticized the MMT for its lack of directionality. Specifically, comparing the differences between reasoning problems involving "If X then Y" and the ones involving "X only if Y". Namely, people are more likely to draw backward inferences (inferences from Y or ¬Y as a minor premise) in the "only if" case and they are more likely to draw forward inferences (inferences from X or ¬X as a minor premise) in the case of "if...then". Assuming a directionality from the antecedent variable to the consequent variable, the only reachable conclusions are the ones following that direction. For example, having only the model X Y in the mental representation supports concluding Y from X (MP), but not X from Y (AC).

2.2 Suppositional Theory

The Suppositional Theory assumes that two systems of reasoning are used in order to make a conclusion for a given reasoning problem, called System 1 and System 2. System 1 describes the heuristic or pragmatic inferences, whereas System 2 is dedicated to abstract rule-based reasoning [4]. The influence of each system depends from one individual to another in terms of belief-based vs. logic-based responses.

Evans and Over [4] discuss early advancements towards the invalidity of the material implication interpretation of indicative conditionals, as primarily done by Ramsey [15]. He suggests that when people reason about a conditional "If X then Y", they are judging the probability of Y, *supposing* X is true, i.e. P(Y|X), widely known as *The Ramsey Test*, which leads to the individual's degree of belief in the conditional itself. Building up on this, System 1 is capable of only generating conclusions that follow *directly* from an individual's belief in the conditional, leading to accepting only MP inferences, unless additional conditional premises are added through pragmatic implicature and the MP inference schemata is applied to them.

System 2, on the other hand, can accept MT by a suppositional line of proof [12]. The suppositional proof starts by supposing that X is true, which combined with the given conditional rule, "If X then Y", leads to the conclusion that Y is true (MP inference). However, this contradicts the minor premise of MT, $\neg$Y, therefore the supposition must be false, so $\neg$X must be true. This leads to the acceptance of the MT inference form. Starting the suppositional proof with Y, combined with the converse, "If Y then X" (assuming that the converse of the conditional has been added by pragmatic implicature), leads to accepting the DA inference form.

2.3 Formalizations

Oberauer [12] provides formalizations of the MMT and Suppositional Theory as multinomial processing trees (MPTs). The MPT can be interpreted as a binary decision diagram with parameters on the edges. In the case of MMT (Fig. 1), the decisions are whether a human reasoner will add a mental model to their representation. For the Suppositional Theory (Fig. 2), the parameters describe the degree of belief in a conditional and/or its converses and inverses.

The leafs represent combinations of inference forms that are *accepted* if that specific path has been taken. It can be noted that there are combinations that the theories cannot express. In our setup we are not interested in the acceptance, but rather we aim to model *individuals* who provide *probabilistic endorsements* in the range 0-100. Towards that goal, we derived expressions from the MPTs for each inference form *separately*, by summing up and simplifying the probabilities of all possible paths leading to accepting said inference form.

For the MMT, Oberauer [12] presents three different formalization variants, and two variants for the Suppositional Theory. For each theory, we focus *only* on the variant that is best performing, so we adapted all five variants to model probabilistic data, examined their predictive performance and picked the best ones. In the following, we introduce the detailed technical background and endorsement expression derivation solely for them.

Mental Model Theory Formalizations Oberauer [12] presents three formalization variants for the MMT: *original*, *deductive* and *with directionality*. The best performing variant is *MMT with Directionality*.

The *original* variant is built directly on equations provided by Schroyens et al. [18]. The *deductive* variant is a special case of the original one, where the model X¬Y is impossible to be added to the mental representation.

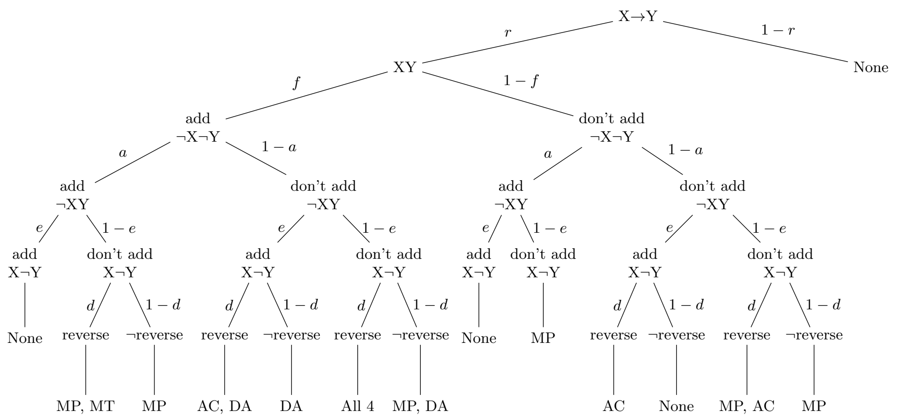


Fig. 1: Formalization of the MMT with Directionality for the conditional "If X then Y" [12]. The parameters r, f, a, e, d take on values in the interval $[0, 1]$, indicating the probability of taking the respective decision path in the model. The leafs represent the responses. "reverse": Reversing the directionality; "¬reverse": Not reversing the directionality.

The third variant is inspired by the Evans' criticism [3] on the lack of directionality, which we elaborated on in Section 2.1. In this formalization, Oberauer [12] adds a parameter that allows the possibility for the directionality to be reversed, in order to overcome that limitation.

We derived the following inference form endorsement expressions for *MMT with Directionality* based on the MPT, shown in Fig. 1:

MP: $r \cdot (1 - e)$ **DA:** $r \cdot f \cdot (1 - a)$

AC: $r \cdot (1 - a) \cdot d$ **MT:** $r \cdot f \cdot (1 - e) \cdot d$

The parameters describe the probabilities of adding models to the representation. r is the probability of adding the model X Y, f for the model ¬X¬Y, a for ¬X Y, e for X ¬Y. Additionally, the parameter d describes the probability of reversing the directionality.

In total, MMT with Directionality needs five parameters to model one *task*.

Suppositional Theory Formalizations Evans and Over [4] do not explicitly specify how the two systems interact, therefore Oberauer [12] provides two variants – an *Exclusive* and a *Sequential* one. The best performing variant is the Sequential one.

The *Exclusive* variant assumes that *either* System 1 *or* System 2 is used exclusively when reasoning.

The *Sequential* variant (Fig. 2) assumes that the two systems work sequentially. After the individual determines their degree of belief in the conditional X→Y, System 1 takes over and has the possibility of adding the converse, Y→X, and inverse, ¬X→ ¬Y by pragmatic implicature. By combining the two, it can arrive to the contraposition, ¬Y→ ¬X. Once System 1 is done with adding conditionals, it derives all the conclusions following from them. Next, System 2 takes the conditionals System 1 has added and with a certain probability and applies the procedure for suppositional proof. Two different suppositional proofs can take place. One supposes the truth of X and the original conditional and the other supposes the truth of Y and the converse.

We derived the following inference form endorsement expressions for *Suppositional Sequential* based on the MPT, shown in Fig. 2:

$$\textbf{MP: } b \qquad \textbf{DA: } b \cdot (c \cdot s^* \cdot (1-i) + i)$$

$$\textbf{AC: } b \cdot c \qquad \textbf{MT: } b \cdot ((1-s) \cdot (2 \cdot c \cdot i - c^2 \cdot i^2) + s)$$

The parameter b describes the belief in the conditional X→Y, c is the probability of the converse to be added (Y→X) and i, the inverse (¬X→ ¬Y). s is the probability of the suppositional proof using X and the original conditional to succeed and s^* for the proof using Y and the converse.

In total, Suppositional Sequential needs five parameters to model one *task*.

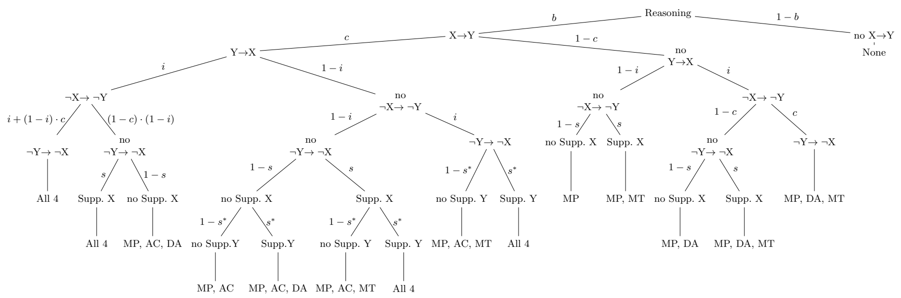


Fig. 2: Formalization of the Suppositional Theory (Sequential Variant) for the conditional "If X then Y" [12]. The parameters b, c, i, s, s^* take on values in the interval $[0, 1]$, indicating the probability of taking the respective decision path in the model. The leafs represent the responses.

2.4 Oaksford-Chater Probabilistic Model

Oaksford and Chater [11, 9, 10] propose a probabilistic model for conditional reasoning. By using a 2×2 contingency table (Table 4), they represent conditional rules, where $a = \mathrm{P(X)}$ and $b = \mathrm{P(Y)}$, probabilities of the antecedent and consequent, respectively and $\epsilon = \mathrm{P}(\neg\mathrm{Y}|\mathrm{X})$ is the exception parameter.

Table 4: Contingency table for a conditional rule "If X then Y". $a = \mathrm{P(X)}$ - Probability of the antecedent; $b = \mathrm{P(Y)}$ - Probability of the consequent; $\epsilon = \mathrm{P}(\neg\mathrm{Y}|\mathrm{X})$ - Probability of the exception.

	Y	¬Y
X	$a(1-\epsilon)$	$a\epsilon$
¬X	$b - a(1-\epsilon)$	$(1-b) - a\epsilon$

This model belongs to the new Bayesian paradigm – it assumes that inference form endorsement is proportional to the conditional probability of the conclusion given the minor premise. So, from Table 4, they derived expressions for the inference form endorsements as follows:

$$\textbf{MP: } \mathrm{P(Y|X)} = 1 - \epsilon \qquad \textbf{DA: } \mathrm{P}(\neg\mathrm{Y}|\neg\mathrm{X}) = \frac{1 - b - a\cdot\epsilon}{1-a}$$

$$\textbf{AC: } \mathrm{P(X|Y)} = \frac{a(1-\epsilon)}{b} \qquad \textbf{MT: } \mathrm{P}(\neg\mathrm{X}|\neg\mathrm{Y}) = \frac{1 - b - a\cdot\epsilon}{1-b}$$

The Oaksford-Chater model needs three parameters to model one *task*.

3 The ϵ-MMT: Mental Models and Probabilities

In previous work [20, 21] we proposed a probabilistic cognitive model for conditional reasoning that is based on a mental model approach and showed that it is a valuable competitor among Bayesian models. We represent conditionals using mental models, interpret them as *possible worlds*, allowing for an application of a probabilistic assignment as described by ϵ-semantics [14].

ϵ-semantics is a 'formal framework for belief revision', as introduced by Pearl [14]. Belief statements are interpreted as high probability statements and newly available evidence leads to belief revision. The following definition [14] lays the basis for the probabilistic aspect of ϵ-MMT:

> Let L be the language of propositional formulas, and let a *truth-valuation* for L be a function t, that maps the sentences in L to the set $\{1,\ 0\}$, (1 for 'true', 0 for 'false'). To define a probability assignment over the sentences in L, we regard each truth valuation t as a world w and define $\mathrm{P}(w)$ such that $\sum_w \mathrm{P}(w) = 1$. This assigns a probability measure to each sentence l of L.

ϵ-MMT takes the conditional's mental model representation of *all* propositions' truth state combinations, meaning that the material implication interpretation is abandoned by allowing the X $\neg$Y model to exist. From now on, we will refer to them as *possible worlds*. Given a conditional premise "If X then Y", all possible worlds describing the states of X and Y along with the corresponding probability values assigned to them are shown in Table 5.

Table 5: Representation of a conditional premise "If X then Y" with possible worlds with a probability distribution P assigned to them and values $p_i, i \in (1, 2, 3, 4)$

World	X	Y	P
ω_1	0	0	p_1
ω_2	0	1	p_2
ω_3	1	0	p_3
ω_4	1	1	p_4

We follow the approach of previous accounts in the Bayesian paradigm [11], and interpret inference form endorsements as a conditional probability of the conclusion given the minor premise.

$$\mathrm{P}(\beta|\alpha) = \frac{\mathrm{P}(\alpha \wedge \beta)}{\mathrm{P}(\alpha)} \tag{1}$$

By applying the definition of conditional probability, as shown in Eq. 1, to Table 5, we derive the following inference form endorsement expressions:

$$\textbf{MP:}\ \mathrm{P}(\mathrm{Y}|\mathrm{X}) = \frac{p_4}{p_3 + p_4} \qquad \textbf{DA:}\ \mathrm{P}(\neg\mathrm{Y}|\neg\mathrm{X}) = \frac{p_1}{p_1 + p_2}$$

$$\textbf{AC:}\ \mathrm{P}(\mathrm{X}|\mathrm{Y}) = \frac{p_4}{p_2 + p_4} \qquad \textbf{MT:}\ \mathrm{P}(\neg\mathrm{X}|\neg\mathrm{Y}) = \frac{p_1}{p_1 + p_3}$$

The ϵ-MMT model needs three free parameters that are bound by the sum $\sum_i p_i = 1$ to model one *task*. The number of parameters needed to model an *individual* depends on the number of tasks.

4 Benchmark and Predictive Modeling Results

We implemented a benchmark within the framework CCOBRA. The main goal is to examine the cognitive models' ability to predict a value in the range 0-100, by taking into consideration *how close* the model's prediction is to the individual's true answer. We evaluate a model's performance by calculating the mean individual *absolute difference* between a prediction and the corresponding true answer. A lower value indicates better performance.

The cognitive models are first exposed to existing data – a *training set*, from which it can learn about the individual's reasoning behavior. Then, the model is challenged with predicting answers in unseen data – a *test set*. Here, we use Singmann et al.'s [19] experimental data for both training and testing. In this case, the CCOBRA framework uses a leave-one-out cross-validation method – models are fit on every participant, except the one whose answers should be predicted. The same is done for each participant. The models are fit to the training data by optimizing their parameter values in order to minimize the absolute difference between the prediction and the true answer. That was done using Python's `scipy.optimize.minimize`[3] with the method Sequential Least Squares Programming (SLSQP), which allows for constrained minimization.

In order to set a performance lower bound, our benchmark has a Random model, which simply gives a random value between 0-100 as a prediction. Since the models do not adapt to an individual, the upper bound is set by an empirical Most Frequent Answer (MFA) model, which gives the *median* of the responses as a prediction.

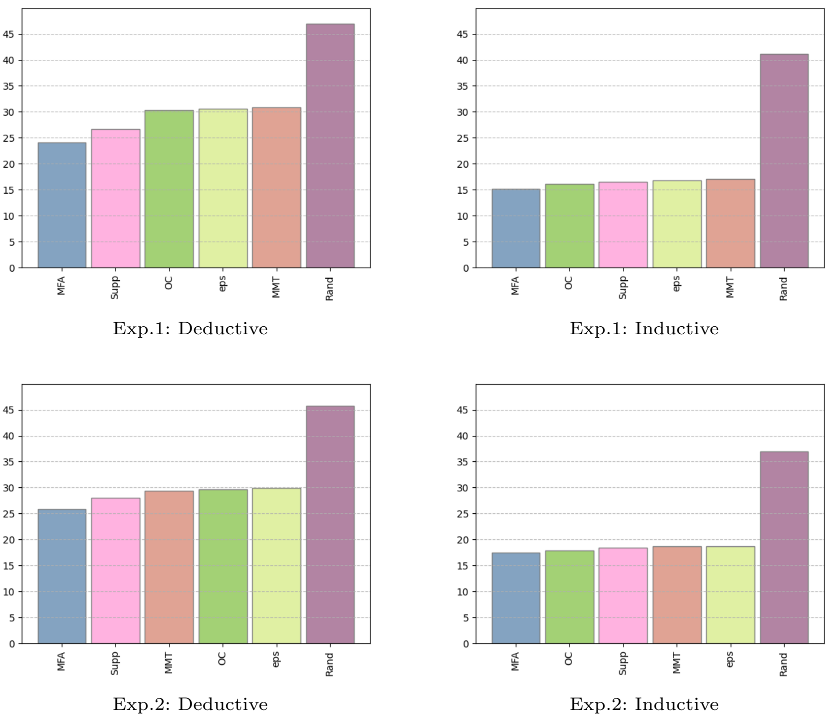
The results are shown in Figure 4. In the deductive case, the Suppositional Sequential model has an outstanding performance for both experiments (Exp. 1: 26.72, Exp. 2: 27.98), whereas the other models' perform similarly, reaching an average absolute difference of 30.57 (Exp. 1) and 29.70 (Exp. 2). In the inductive case, however, the Bayesian Oaksford-Chater model takes the lead with an absolute difference of 16.08 in Exp. 1 and 17.90 in Exp. 2. Once again, a similar performance among the other models can be seen, having an average absolute difference of 16.82 (Exp. 1) and 18.57 (Exp. 2).

We have two mental model based approaches, a probabilistic endorsement adaptation of Oberauer's [12] formalization MMT with Directionality, and our proposed model ϵ-MMT that follows the Bayesian paradigm. ϵ-MMT outperforms MMT with Directionality in Exp. 1, but performs slightly worse in Exp. 2. Though, the difference between the two models is no more than 0.55 in any scenario. It is noteworthy that the total number of free parameters that each model needs for one task is different. Here, ϵ-MMT takes the lead with only three parameters, in contrast to five for MMT with Directionality.

We continue our analysis as we look into the models' parameter values to learn more about the difference between deductive and inductive instructions. For that purpose the main focus is on ϵ-MMT's p_2 and p_3, Oaksford-Chater's ϵ and MMT with Direcionality's a and e parameters. Their values for each instruction and conditional type are shown in Table 6.

Material implication states that a conditional "If X then Y" is false when X is true and Y is false. These three models are able to recognize this state through their parameters for counterlogical conditionals through low values under deductive instructions. More specifically, ϵ-MMT's p_3 parameter describes the belief in the world ω_3, where X = 1 and Y = 0. Oaksford-Chater's ϵ parameter describes the conditional probability P(¬Y|X). MMT with Directionality's e parameter describes the probability of the mental model X ¬Y to be added to the mental representation. In both experiments, these parameters have an increase in value for individuals that had inductive instructions showing the material implication interpretation under deductive instructions.

[3] `https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.minimize.html`



Exp.1: Deductive

Exp.1: Inductive

Exp.2: Deductive

Exp.2: Inductive

Fig. 4: Predictive modeling benchmark results. Means of all individual absolute differences between the true participants' answers and the models' predictions. MFA and Random are the upper and lower bound, respectively. Supp – Suppositional Sequential, OC – Oaksford-Chater, eps – ϵ-MMT, Rand – Random.

Another point of interest is the influence of alternatives and disablers, under *inductive instructions*, when background knowledge is taken into consideration. In the following, we use X for the antecedent and Y for the consequent ("If X then Y"). The presence of many alternatives means that Y occurs even if X has not. ϵ-MMT's p_2 parameter describes the belief in that world (ω_2) and MMT with Directionality's a parameter describes the probability of the corresponding mental model ($\neg$X Y) to be added to the representation. In both experiments, for prological conditionals the values of these parameters are higher in comparison to counterlogical, reflecting the influence of alternatives. On the other hand, the presence of many disablers means that even if X has happened, Y has not. ϵ-MMT's p_3 parameter describes the belief in that world (ω_3), Oaksford-Chater's ϵ parameter describes the conditional probability for that scenario, P($\neg$Y|X) and MMT with Directionality's e parameter describes the probability of the corresponding mental model (X $\neg$Y) to be added to the representation. Similarly, in both experiments for counterlogical conditionals, these parameter values are higher in comparison to prological ones, pointing to the effect of disablers.

Table 6: Medians of the models' parameter values per instruction and conditional type. ϵ-MMT: Probabilities of possible worlds, $p_2 = \mathrm{P}(\omega_2)$, $p_3 = \mathrm{P}(\omega_3)$, see Table 5; Oaksford-Chater: Exception parameter $\epsilon = \mathrm{P}(\neg\mathrm{Y}|\mathrm{X})$; MMT with Directionality: Probabilities of adding mental models in representation, a for the model $\neg$X Y and e for X $\neg$Y.

Exp.	Instructions	Conditional Type	ϵ-MMT		Oaksford-Chater	MMT with Dir.	
			p_2	p_3	ϵ	a	e
1	Deductive	Counterlogical	0.09	0.05	0.14	0.08	0.03
		Prological	0.20	0.03	0.11	0.19	0.00
	Inductive	Counterlogical	0.01	0.22	0.43	0.02	0.40
		Prological	0.19	0.00	0.05	0.15	0.03
2	Deductive	Counterlogical	0.37	0.08	0.15	0.37	0.04
		Prological	0.42	0.03	0.06	0.44	0.00
		Neutral	0.14	0.04	0.13	0.12	0.06
	Inductive	Counterlogical	0.15	0.11	0.34	0.18	0.24
		Prological	0.30	0.03	0.08	0.38	0.03
		Neutral	0.03	0.10	0.24	0.05	0.19

5 Discussion and Conclusion

Research approaches have been divided between emphasizing logical validity and focusing on content and background knowledge. Those are different types of reasoning that one single mechanism cannot cover, leading to arguments that one of the two approaches is wrong [19, 9, 17]. Singmann and Klauer [19] show a double dissociation of the two types of instructions. To further analyze this, we adapted MPT formalizations of cognitive theories such that they can now represent experimental data with probabilistic endorsements. Then, we compared their *predictive* performance to an established Bayesian model and a newly proposed probabilistic approach based on mental models. We showed that after exposing conditional models to a training data set, they are capable of generating predictions on unseen data in the range 0-100. In previous work [21] we considered experiments where participants received inductive instructions and the models reached a mean deviation of ca. 17, which is what we encounter now as well. However, we learn that their performance drops to ca. 29 when dealing with deductive instructions. An important finding is that the Suppositional Sequential formalization stands out under deductive instructions, while the performance of the Oaksford-Chater model exceeds that of the others under inductive instructions. With this we show that only one single theory can *not* explain all reasoning process under all circumstances. The difference in performance lays basis for future research to examine the difference in reasoning processes under various conditions through cognitive modeling. Additionally, we showed that ϵ-MMT, Oaksford-Chater and MMT with Directionality are capable to account for the effect of instruction, by showing the material implication interpretation under deductive instructions. More detailed research is necessary for Suppositional Sequential, especially given that it excels for deductive instructions. Additionally, we confirmed our finding [21] that ϵ-MMT can successfully account for the influence of disablers and alternatives when background knowledge is taken into consideration. Both ϵ-MMT and MMT with Directionality are based on mental models, have a similar performance and account for instruction and content effect in the same way through their parameter values. The advantage of ϵ-MMT is that it needs only three free parameters in contrast to five. That indicates that the same information can be represented by ϵ-MMT in a less complex and computationally more efficient way, while still preserving the same psychological interpretation and providing valuable insight. As mentioned before, the MPTs for MMT and the Suppositional Theory show that certain combinations of inference forms are lacking from the trees and cannot be accepted according to the theories. Even though we are not looking into combinations of accepted inferences, but rather their separate probabilistic endorsements, the parameter values still depend on the combinations. In future work we would look into how influential this is on the model's predictive performance. Namely, is the lack of combinations hurting the model, or is it accurately describing human reasoning? Furthermore, an even more exciting next step would be to challenge the models to *adapt* to individuals as well. With this we continue the thrilling path of challenging the predictive capabilities of models and gaining insight about the individual reasoning processes under various effects.

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