# Adaptive Signal Filtering Algorithm in Telemedicine

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**Abstract**

The article presents the results of an experiment of filtering a mixture of electrocardiogram signal with superimposed noise, represented by a discrete time series. The work of the signal-to-noise mixture filtering algorithm is considered for the cases of white, gaussian, pink and brown (Brownian) noises. The results of an experiment on finding the optimal filtering parameters in cases with normal and imperfect noise are presented. The algorithm uses wavelet filtering and filtering methods based on Kalman's algorithm. In the course of the work, the effectiveness of the filtering algorithm for medical signals was confirmed.

**Keywords**

Wavelet filtering, Kalman algorithm, SNR, Electrocardiogram filtering

## 1. Introduction

Nowadays, remote medical support gains more popularity, posing new challenges [1, 2]. One of these cases is to provide the transmission accuracy of medical information for correct and early diagnosis. In connection with the above, the task is to ensure a sufficient data transfer rate, as well as to obtain a clean signal, without noise and interference. Filtering is an appropriate method for obtaining a clean signal [19].

This paper presents the results of a numerical experiment of filtering and sufficient information transfer rate problem in the medical industry. This task is considered as part of the methodology for secure data transmission, including signal compression and encryption algorithms in addition to preprocessing. The goal is to conduct numerical experiment with an assessment of the results for the selected metrics and errors obtained in process. It is necessary to understand that there are many standards for the medical data transmission, as well as signal (such as ECG, SCG and flowmetry) filtering techniques.

First, the metrics for evaluating the type of noise and filtering success are considered, then the results are presented.

## 2. Metrics and Methods

Medical signals are represented by both the digital and analog family of signals. As it turns out, there is not much difference in analog and digital filtering methods. It would be more correct to call a digital signal filtering simpler than an analog one, due to the discreteness of the first. In this paper, only digital signals are being considered.

Digital signal processing solves two main problems: detecting and determining the parameters of a noisy signal.

The algorithm, the results of which are presented here, can be called adaptive, since makes its own adjustment depending on the detected type of noise. Let's select the metrics that we will use in the experiment.

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The experiment was carried out for ECG signals based on the Medical Signals Database of St. Vincent University Hospital[2].

## 2.1. Signal to noise ratio

SNR, or Signal to Noise Ratio, (hereinafter SNR) metric will be considered as a metric of the filtering success. Let's represent the noisy signal as the sum of the required signal and noise:

$$y(n) = s(n) + w(n), \tag{1}$$

where $n$ is the measurement number,
$s(n)$ – the required signal,
$w(n)$ – noise,
$y(n)$ – the signal-and-noise mixture is the time series with the number of measurements (nodes) $N$, which is fed to the input of the filtering algorithm.

To assess the effect of noise in the channel on signal transmission, the concept of "signal-to-noise ratio", which is basically the power-of-signal to power-of-noise ratio, is introduced as:

$$SNR = \frac{P_{signal}}{P_{noise}}. \tag{2}$$

If all the components are expressed in decibels, then the formula can be simplified to:

$$SNR_{dB} = P_{signal\_dB} - P_{noise_{dB}} = s(n) - w(n). \tag{3}$$

In all the above cases, $P$ is the average power. Also, the signal-to-noise ratio can be obtained by squaring the ratio of the root mean square values of the signal and noise amplitudes, respectively.

## 2.2. Allan Variance

Variances are used to characterize fluctuations in frequency data. Allan variance is also known as the variance of the difference between the values of the relative readings $y_i$ and $y_{i+1}$, measured at times $t_i = t_0 + i * \tau$ and $t_{i+1} = t_0 + (i+1) * \tau$, respectively (for $M$ measurements):

$$\sigma_y^2(\tau) = \frac{1}{2(M-1)} \sum_{i=0}^{M-1} (Y_{i+1} - Y_i)^2, \tag{4}$$

where $M = \left\lceil \frac{T}{\tau} \right\rceil - 1$,

$$Y_i = \frac{1}{M} \sum_{k=i}^{i+M-1} y_k,$$

[2] PhysioNet. St. Vincent's University Hospital / University College Dublin Sleep Apnea Database, 2021. URL: https://physionet.org/content/ucddb/1.0.0/ (Accessed July 6, 2021)

$y_k = \frac{x_{k+1} - x_k}{\tau_0}$ and $Y_i$ is the $i$-th of $M$ values of the particular frequency values averaged over the measurement (sample) interval (according IEEE Std 1554-2005[3]).

The applications of Allan variations in the problems of detecting noise and chaotic processes in biomedical data give interesting results [15-18].

## 2.3. Hadamard variance

The Hadamard variance is based on the Hadamard transform, which is suitable for measuring frequency stability in the time domain and has a higher resolution than Allan variance. For the frequency series, the Hadamard variation looks like this:

$$H\sigma_y^2(\tau) = \frac{1}{6(M-2)} \sum_{i=0}^{M-2} (Y_{i+2} - 2Y_{i+1} + Y_i)^2, \tag{5}$$

where $Y_i$ - $i$-th frequency value from $M$, averaged over time $\tau$ [3].

Having calculated the variations of $\sigma(\tau)$ for different $\tau$ and plotting their mutual dependence, plotting an approximating curve from its slope, we could make an assumption about the type of noise coming with the signal.

# 3. Numerical experiment

In this article, the following signal processing algorithm is used:

1. Evaluation of the signal-to-noise mixture using the Allan and Hadamard variances.
2. Determining the type of noise presented in the mixture based on these metrics.
3. Selection of the most suitable filtering method based on the type of noise (for white and Gaussian noises in the mixture, approximation is performed using wavelet filtering, for pink and brown noises, the Kalman algorithm with B-splines is selected).
4. Mixture time series direct approximation.

In case of an ideal noise, it will be relevant to maximize the SNR metric by choosing an effective threshold function with a suitable threshold $\lambda$. For the variant of other noises, it is important to find the optimal value of the interval, taken for filtering at one moment in time, with the number of measurements $M$.

Let us impose restrictions on the sampling rate of the considered mixture of signal and noise and on the length of the template:

$$\begin{cases} \nu_i = \nu \\ t_{min} \le t_i \le t_{max} \end{cases}. \tag{6}$$

Sampling rate ($\nu$). To create a template, we need to sample at a specific frequency. Let's take the sampling rate $\nu$ = 100 Hz. This frequency is the most common among the sampling rates of signals in medicine.

Maximum and minimum template length ($t_{max}$ and $t_{min}$). It is assumed that the heart rate is in the range of 40-120 beats per minute. Therefore, the maximum length of the template which includes at least one complete cardiac cycle fell, we take $t_{max}$ = 1.5s. The minimum template length will be $t_{min}$ = 0.5s.

It is required to obtain the maximum increase in the SNR value after the approximation in comparison with the SNR before the time series approximation:

[3] IEEE Recommended Practice for Inertial Sensor Test Equipment, Instrumentation, Data Acquisition, and Analysis, IEEE Std 1554-2005, pp.1-145, 22 Nov. 2013, doi: 10.1109/IEEESTD.2013.6673990.

$$SNR_{\text{approximated\_signal}} - SNR_{\text{noisy\_signal}} = P_{signal\_dB} - (P_{noise\_dB} - P_{approximated\_noise\_dB}) - (P_{signal\_dB} - P_{noise\_dB}) = s(n) - w_{before\_smoothing}(n) - (s(n) - w_{after\_smoothing}(n)) => w_{before\_smoothing}(n) - w_{after\_smoothing}(n) \rightarrow max\ . \quad (7)$$

## 3.1. Filtering signals

Noise was carried out on 25 three-band electrocardiograms of the EDF (European Data Format) format. Fig. 1 shows the 10-second interval of one of three ECG signals.

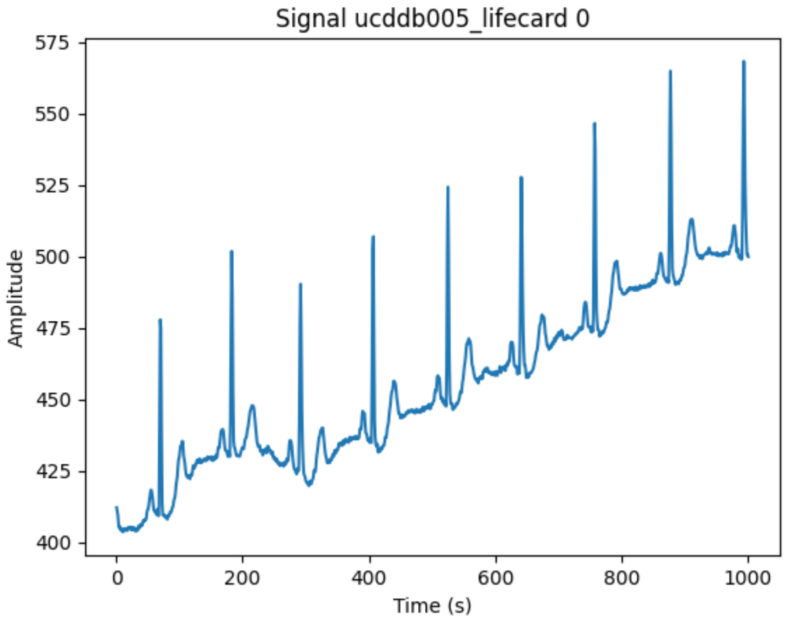


**Figure 1**: ECG signal with a sampling rate of 100 Hz
(The number of measurements is shown horizontally)

An example of applying white noise to this signal can be seen in Figure 2. As a result of overlaying, we get a signal and noise mixture. As a result of finding the variances of Allan and Hadamard (Fig. 3-5), we can determine the type of noise. By the slope of the approximating straight line, we determine the type of the noise. After that, we start filtering.

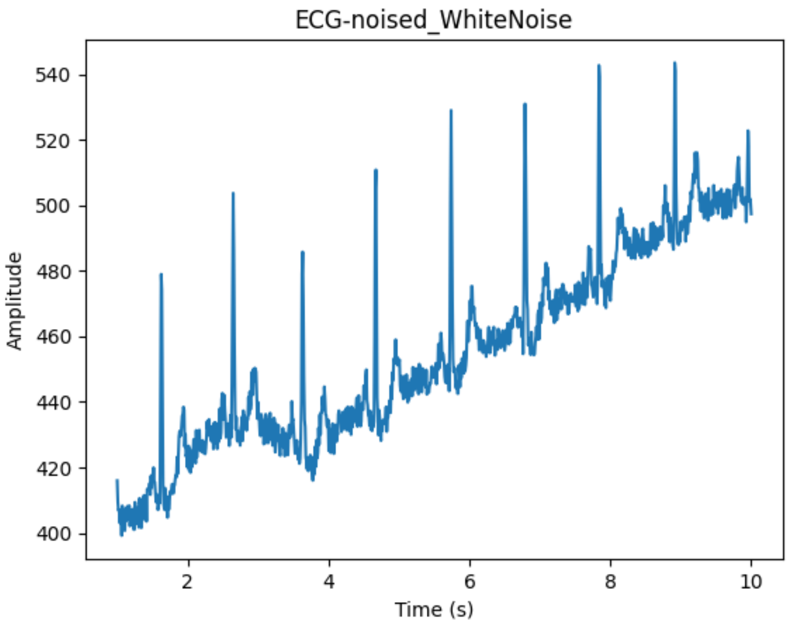


**Figure 2**: Mix of ECG signal with white noise

## 3.2. Wavelet filtering algorithm

The wavelet filtering algorithm is based on the properties of the wavelet decomposition [4-7]. As a threshold, we use a garrote function:

$$T_G(\tilde{d},\lambda) = \begin{cases} \tilde{d} - \frac{\lambda^2}{\tilde{d}}, \text{если } |\tilde{d}| \geq \lambda \\ 0, \text{если } |\tilde{d}| < \lambda \end{cases}. \tag{8}$$

We carry out an experiment with filtering for the following groups of functions: symmelets, coiflets and Daubechies wavelets.

The principle of the filter consists in a sequential discrete wavelet transform, processing of the obtained detailing coefficients by a threshold function and inverse wavelet transform (Fig. 7).

## 3.3. Algorithm based on Kalman filtering

The Kalman filter using B-spline functions is a bit like the sliding window method [8-10].

As a result of using the algorithm and taking into account the SNR metric:

$$\begin{aligned} & w_{before\_smoothing}(n) - w_{after_{smoothing}}(n) \\ & = c * x - w_{after\_smoothing}(n) - (c * x - w_{before\_smoothing}(n)) \\ & = \sum_{i=1}^{M} V(n_i)B_{i,k}(n) + w_{before\_smoothing}(n) \\ & - \sum_{i=1}^{M} V(n_i)B_{i,k}(n) - w_{after\_smoothing}(n). \end{aligned} \tag{9}$$

It is necessary to find the optimal width of the interval $M$ in order to obtain the maximum increase in the value of the signal-to-noise metric:

$$\begin{cases} y(n) = \sum_{i=1}^{M} V(n_i) B_{i,k}(n) + w(n) \\ w_{before\_smoothing}(n) - w_{after\_smoothing}(n) \to \max_{M}, \\ M \leq 150 \end{cases} \tag{10}$$

where $n_i$ – is the position of the $i$-th dimension,

$V(n_i)$ – control points, the position of which is determined depending on the values of the approximation errors,

$M$ – total number of measurements based on sampling rate and pattern width, $M \leq 150$,

$B_{i,k}(n)$ – B-spline function of order k associated with measurements $n_i,\ldots,n_{i+k}$.

This type of processing of the signal-to-noise mixture consists in the sequential execution of the Kalman filtering algorithm for intervals containing $M$ measurements [11]. One can read more about the intricacies of using the Kalman filter for processing biomedical data in publications [12-14].

## 4. Results

As a result of determining the type of noise stage, it is possible to recognize the type of noise in the mixture (Fig. 3-5).

One can notice a general trend with a decrease in the difference between the signal-to-noise ratio before and after filtering with an increase in the value of the threshold function. The value of the threshold $\lambda$, which is in the vicinity of the value 5, can be considered optimal. Columns in Fig. 6 and 8 (that's the results of developed software processing) one by one:

1. Noise type, 2. Wavelet view, 3. Transformation level, 4. Threshold function, 5. Threshold value, 6. SNR of the mixture before filtering, 7. SNR of the mixture after filtering, 8. SNR difference.

| Tau | ADEV | HDEV |
|---|---|---|
| 1 | 1.6536 | 1.7427 |
| 2 | 0.8188 | 0.86152 |
| 4 | 0.42201 | 0.44505 |
| 8 | 0.2099 | 0.22143 |
| 16 | 0.10645 | 0.11014 |
| 32 | 0.064233 | 0.053651 |
| 64 | 0.079976 | 0.04976 |
| 128 | 0.10903 | 0.11134 |
| 256 | 0.067495 | 0.07055 |

**Figure 3**: Values of Allan and Hadamard variations depending on $\tau$ for example signal

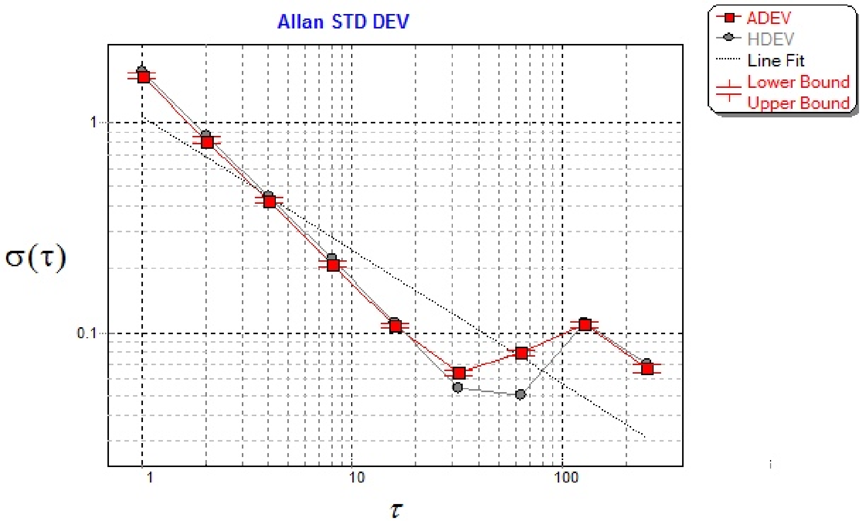


**Figure 4**: Allan and Hadamard variances for a mixture of sinusoidal signal with white noise

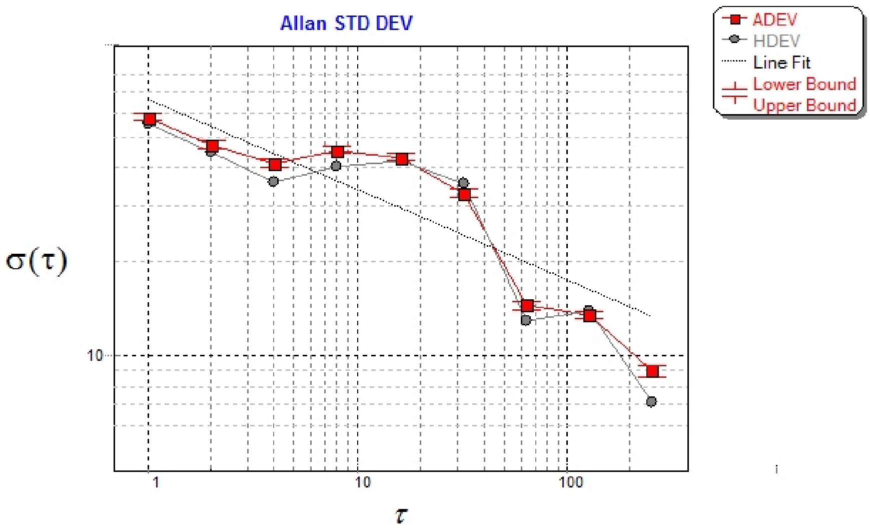


**Figure 5**: Allan and Hadamard variances for mixture of ECG signal with white noise

Research practically shows that the 3rd level of wavelet transform gives the largest number of high smoothing results. The best values for white and Gaussian noise are shown in Fig. 8.

| noise type | wavelet | level | trshld | treshold value | SNR before | SNR after | SNR difference |
|---|---|---|---|---|---|---|---|
| WhiteNoise | coif1 | 3 | std | 15 | 20.322586886449 | 21.769193144616075 | 1.4466062581670762 |
| WhiteNoise | coif1 | 3 | std | 16 | 20.322586886449 | 21.203792533237518 | 0.8812056467885192 |
| WhiteNoise | coif1 | 3 | std | 17 | 20.322586886449 | 20.6983128838431 | 0.3757259973941025 |
| WhiteNoise | coif1 | 3 | std | 18 | 20.322586886449 | 20.60901713422053 | 0.2864302477715306 |
| WhiteNoise | coif1 | 3 | std | 19 | 20.322586886449 | 19.951618970018373 | -0.3709679164306259 |
| WhiteNoise | coif1 | 3 | std | 20 | 20.322586886449 | 19.77512376637597 | -0.5474631200730293 |
| WhiteNoise | coif1 | 3 | garrote | 1 | 20.322586886449 | 20.883313386177743 | 0.5607264997287444 |
| WhiteNoise | coif1 | 3 | garrote | 2 | 20.322586886449 | 22.07042605900597 | 1.7478391725569722 |
| WhiteNoise | coif1 | 3 | garrote | 3 | 20.322586886449 | 23.40962059068779 | 3.0870337042387916 |
| WhiteNoise | coif1 | 3 | garrote | 4 | 20.322586886449 | 24.491381611978046 | 4.168794725529047 |
| WhiteNoise | coif1 | 3 | garrote | 5 | 20.322586886449 | 24.89811946003185 | 4.575532573582851 |
| WhiteNoise | coif1 | 3 | garrote | 6 | 20.322586886449 | 24.578796737576386 | 4.256209851127387 |

**Figure 6**: Example part of the generated tables with the numerical results of the experiment of filtering the signal and noise mixture with a wavelet filter

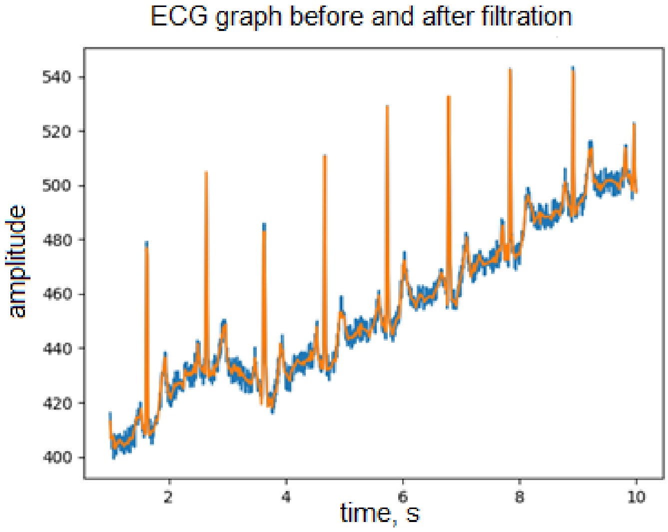


**Figure 7**: Graph of the ECG signal and noise mixture before (blue) and after filtration (orange)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| GaussNoise | coif1 | 3 | garrote | 5 | 15.647455279740793 | 20.709726152091758 | 5.062270872350965 |
| WhiteNoise | coif1 | 3 | garrote | 5 | 20.322586886449 | 24.89811946003185 | 4.575532573582851 |

**Figure 8**: Best results of filtering (part of table: 1. Noise type, 2. Wavelet view, 3. Transformation level, 4. Threshold function, 5. Threshold value, 6. SNR of the mixture before filtering, 7. SNR of the mixture after filtering, 8. SNR difference)

Let's overlay the brown noise (Figure 9) and find the signal-to-noise ratio (Figure 10). $D[x] = \sigma_w^2 = \frac{10^{-6}}{12}, -\frac{10^{-3}}{2} \leq w(n) \leq \frac{10^{-3}}{2}$ is an optimal Brownian noise dispersion added to the signal. Under the sampling rate in the table in Fig. 10 refers to the increase / decrease the ratio of the original sampling rate. The most optimal coefficient is 1, with the value of covariance $\delta = 10^{-4}$. Such conditions are most acceptable for all values of the interval M, but such parameters give the greatest improvement in the signal-to-noise ratio for $M = 99$. The results of the approximation are shown in Fig. 11.

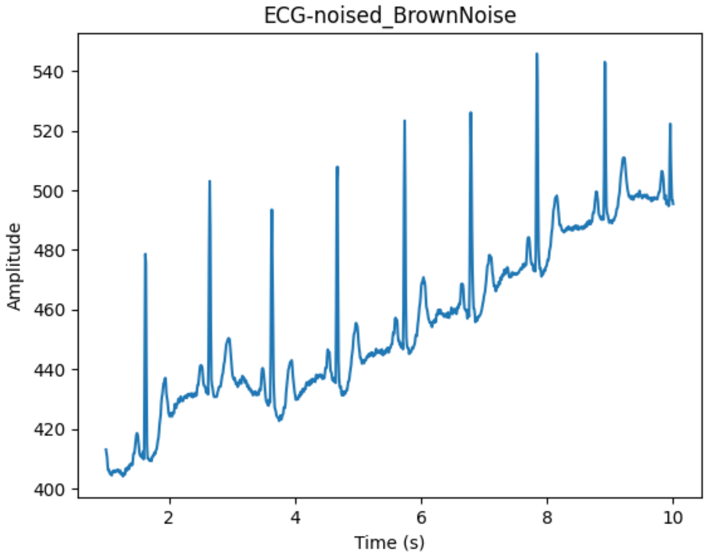


**Figure 9**: Mix of ECG signal with Brown noise

| noise type | D | sampling rate | cov ratio | SNR before | SNR after | SNR differ. | M |
|---|---|---|---|---|---|---|---|
| BrownNoise | 8.333333333333333e-08 | 1 | 0.0001 | [16.52179196] | [19.52179311] | [3.00000115] | 97 |
| BrownNoise | 8.333333333333333e-08 | 1 | 0.0001 | [16.52179196] | [19.52338436] | [3.00159230] | 98 |
| BrownNoise | 8.333333333333333e-08 | 1 | 0.0001 | [16.52179196] | [19.52618960] | [3.00439764] | 99 |
| BrownNoise | 8.333333333333333e-08 | 1 | 0.0001 | [16.52179196] | [19.52048738] | [2.99869542] | 100 |
| BrownNoise | 8.333333333333333e-08 | 1 | 0.0001 | [16.52179196] | [19.52179196] | [2.97650117] | 101 |

**Figure 10**: Part of the table (screenshot from developed software) with the results of filtering the signal and noise mixture by the method based on the Kalman algorithm.
Results by columns: 1. Noise type, 2. Dispersion, 3. Sampling rate, 4. Covariation ratio, 5. SNR of the mixture before filtering, 6. SNR of the mixture after filtering, 7. SNR difference, 8. Interval size.

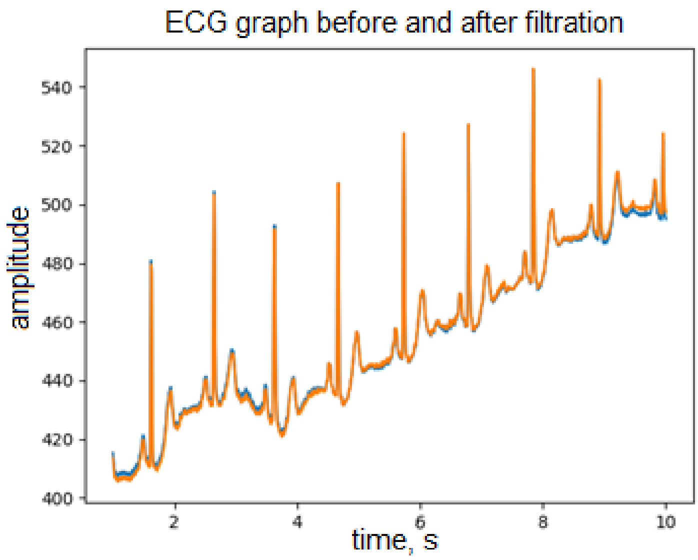


**Figure 11**: Graph of the ECG signal and noise mixture before (blue) and after filtering by Kalman (orange)

## 5. Conclusions

The article presents the results of two filtering algorithms performed depending on the type of noise superimposed on the signal. An increase in the signal-to-noise ratio after filtering the data was confirmed.

The experiment carried out in this article makes it possible to filter the real signal in the form of an ECG using wavelet transforms and threshold functions, as well as the Kalman filter.

Thus, the three-step technique, which includes the preprocessing of the signal described in this article, the compression algorithm and the encryption algorithm, can be considered as one of the methods for safe and fast signal transmission in telemedicine.

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