



**D3 - Methodology for reconciling the historical annual CO<sub>2</sub> aviation emissions estimates based on the actual fuel burn information as provided to EUROCONTROL by volunteer aircraft operators**


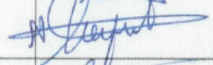
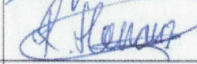

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# 1 INTRODUCTION

## 1.1 Purpose and scope

On 19 November 2008 the European Parliament and the Council of the European Union adopted Directive 2008/101/EC (hereafter referred to as "the Directive") amending Directive 2003/87/EC so as to include aviation activities in the European Union Emissions Trading Scheme (EU ETS).

On 30 December 2008, the European Community and the European Organisation for the Safety of Air Navigation (EUROCONTROL) concluded a cooperation agreement for the provision of support by EUROCONTROL to the European Commission for the inclusion of aviation in the EU ETS. One of the tasks for which EUROCONTROL is providing support is the estimation of "historical aviation CO<sub>2</sub> emissions". Historical aviation CO<sub>2</sub> emissions is to be understood as the average of the annual CO<sub>2</sub> emissions in the calendar years 2004, 2005 and 2006 from aircraft performing an aviation activity included in the EU ETS, and will serve as a basis to set the CO<sub>2</sub> emissions cap for aviation. Considering the financial implications of the total quantity of allowances to be allocated to the aviation sector, it is essential to ensure that the estimation of the historical aviation CO<sub>2</sub> emissions is of the highest possible quality.

EUROCONTROL has established a process for the estimation of the annual historical aviation CO<sub>2</sub> emissions in the EU-27 that consists of the following steps:

- a. establishment of a process for the estimation of the annual historical aviation CO<sub>2</sub> emissions in the EU-27 relying on the air traffic management information available in EUROCONTROL;
- b. calculation of the 2004, 2005, and 2006 annual historical aviation CO<sub>2</sub> emissions estimates based on the above process;
- c. collection of actual fuel burn information from volunteer aircraft operators (AOs);
- d. analysis of the fuel burn information and establishment of a methodology for reconciling the historical annual CO<sub>2</sub> aviation emissions estimates (phase b) relying on this actual fuel burn information;
- e. adjustment of the 2004, 2005, and 2006 annual historical aviation CO<sub>2</sub> emissions estimates based on the above reconciliation methodology for EU-27;
- f. establishment of a methodology for the reassessment, upon the extension of the EU ETS to other states, of the annual historical aviation CO<sub>2</sub> emissions.

The process established by EUROCONTROL within phase (a) has been reviewed in the technical report ref. INX-ETS-TR-09-01 "Report on the review of the process for the estimation of the historical annual CO<sub>2</sub> aviation emissions in line with the Directive prescriptions" and amended by EUROCONTROL according to the recommendations provided therein; and the correctness of the calculation of the 2004, 2005, and 2006 annual historical aviation CO<sub>2</sub> emissions estimates carried out in accordance with the resulting process has been verified within report ref. INX-ETS-TR-09-02 "Report on the validation of the calculation of the 2004, 2005, and 2006 historical annual CO<sub>2</sub> aviation emissions estimates based on the process reviewed in D1".

However, the methodology used for these calculations, which is based on EUROCONTROL implementation of the ANCAT-3 methodology and on the air traffic data available at EUROCONTROL, is not able to account for certain factors that have an influence on the fuel consumed, hence on the CO<sub>2</sub> emissions. By using actual fuel burn data, it is possible to estimate the influence of these factors in order to adjust the theoretical calculations and therefore improve the accuracy of the assessment, thereby

reducing the risk of underestimating or overestimating the total emissions for the 2004-2006 period. With the aim to obtain the best possible estimate of historical CO<sub>2</sub> emissions, EUROCONTROL has requested actual fuel burn data from different aircraft operators. The present document proposes a methodology to make use of the actual fuel burn information collected by EUROCONTROL in order to adjust the calculation and provide an estimation of historical emissions as accurate as possible.

The purpose of the present document is to define a methodology for adjusting the annual historical aviation CO<sub>2</sub> emissions estimates (phase b), based on the analysis of the actual fuel burn information provided by Aircraft Operators (AOs). More specifically, the objectives of the document are:

- to identify those factors that have an impact on the deviation of actual emissions from theoretical estimated CO<sub>2</sub> emissions;
- to assess and quantify the impact of such factors;
- to provide an analysis of the statistical significance of the calibration data, which will depend on the size and characteristics of the sample, in order to minimise the risk of overestimating or underestimating the correction factors due to a biased sample; and
- to define a methodology for the adjustment of the theoretical estimated emissions in order to get a result with optimal accuracy for total historical emissions.

## 1.2 Structure of the document

The document is organised as follows:

- Section 2 provides an overview of those phenomena not accounted for by EUROCONTROL's methodology that are likely to have an influence on the CO<sub>2</sub> emissions of a particular flight.
- Section 3 presents the proposed process for identifying the correction factors by making use of the calibration data. It describes the methodology for analysing the statistical significance of such calibration data, the proposed approach for the adjustment of the theoretical calculation and the process to implement the calculated correction factors and recalculate historical CO<sub>2</sub> emissions.
- Section 4 summarizes the main aspects of the proposed reconciling methodology.
- Annexes I to III provide additional details on the mathematical implementation of the process.

## 1.3 References

- [1] Directive 2008/101/EC of the European Parliament and of the Council of 19 November 2008 amending Directive 2003/87/EC so as to include aviation activities in the scheme for greenhouse gas emission allowance trading within the Community.
- [2] EUROCONTROL Price Enquiry No. 09-110224-E. Technical specification.
- [3] Review of the EUROCONTROL process for the estimation of the historical annual CO<sub>2</sub> aviation emissions in line with the Directive prescriptions. Innaxis-UPM. Ref. INX-ETS-TR-09-01.
- [4] Verification of the calculation of the 2004, 2005 and 2006 historical annual CO<sub>2</sub> aviation emissions estimates based on the process reviewed in D1. Innaxis-UPM. Ref. INX-ETS-TR-09-02.
- [5] ECAC. Recommendation ECAC/27-3 Methodology for emissions calculation.
- [6] EUROCONTROL. Calculation of Emissions by Selective Equivalencing (C.A.S.E.). Edition 1.0, 25 November 2003.

## 2. ASSESSMENT OF INFLUENCING FACTORS

The methodology used by EUROCONTROL is based on the implementation of the ANCAT-3 methodology and on the air traffic data available at EUROCONTROL. It does not take into account a set of relevant parameters that nevertheless have an impact on fuel consumption. Among these factors, we can mention:

- actual take-off weight,
- non optimal flying altitude;
- TMA holding,
- meteorological conditions,
- actual taxi times,
- engines derating level;
- fuel consumed by the Auxiliary Power Unit (APU).

The following picture represents the different factors not accounted for in the ANCAT 3 methodology:

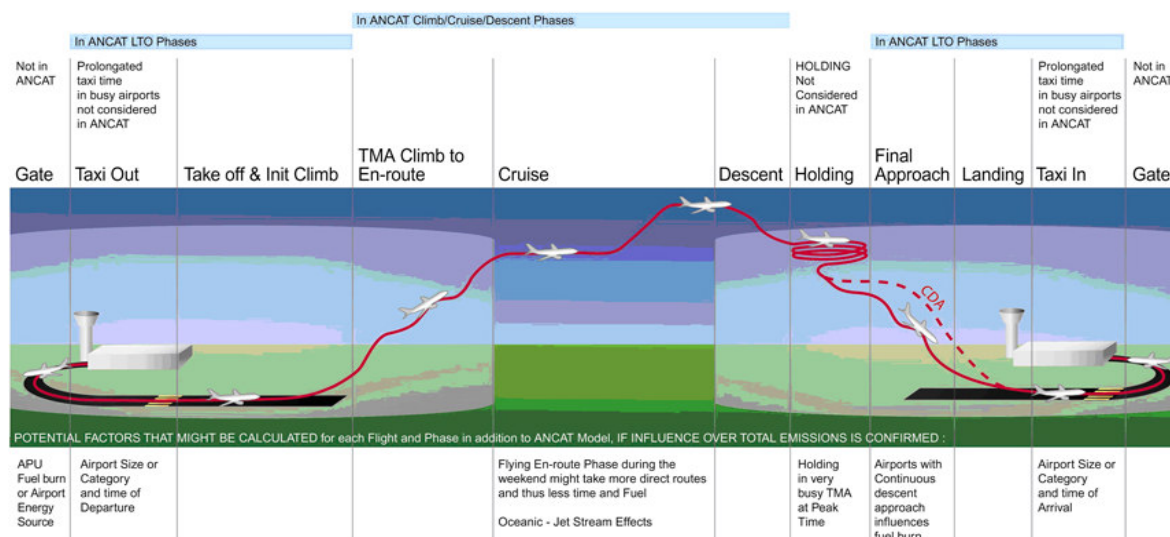


Figure 1. Flight phases and related fuel burn and CO<sub>2</sub> emissions with potential influencing factors (source: EUROCONTROL)

The sample data received by the AOs comprise the fuel consumption from a gate-to-gate perspective; and thus, allows to quantify the main factors that influence the fuel consumption.

For short range flights, the main sources of inaccuracy are normally the TMA and the ground part of the flight, while for long range flights the flight conditions and the actual payload become the most influencing factors. As it is shown in the picture, factors such as gate phases, holding or extended taxiing are not accounted for in ANCAT. However, given that the AOs fuel consumption data do comprise these, with the proposed methodology they will be accounted for in total fuel consumption terms.

In order to consider these factors, the recommended procedure is to adjust the theoretical consumption applied on real traffic data with real fuel data obtained from aircraft operations. The adjustment needs to follow certain principles to make sure the procedure has a strong scientific basis:

- It is necessary to determine whether the samples are representative of the whole population of data or, on the contrary, if the data samples show any bias or gap. While some of the above factors are simply expected to add a random noise to the theoretical fuel consumption, other factors are likely to introduce a bias in the theoretical estimation if they are not properly taken into account. It is therefore necessary to ensure that the sample is representative of the whole population of flights as far as these influencing factors are concerned: if, for example, the level of congestion of the origin and/or destination airports are found to have an influence on the correction factors, it will be necessary to ensure that the sample contains a mix of congested/non-congested airports in a proportion similar to that of the totality of flights.
- After checking the representativeness of the sample, it is also important to assess the accuracy of the new adjusted factors, which will be evaluated by calculating the confidence level.
- The adjustment needs to be performed at the proper scale to account for all factors listed above (meteorological conditions, airline strategy, taxiing, and holding). Maintaining the analysis at a macro-level and checking the representativeness of the sample ensures that the fuel data sample is descriptive enough to provide an overall correction that includes all different factors present in the European aviation network, since the actual data will capture the influence of different airports (congested, non-congested), meteorological conditions, different airlines strategies, route deviations or different holding patterns.

It is worth noting that:

1. The correction of the theoretical model is intimately linked to the quality of the fuel data samples available and the correction cannot be designed without taking into account the size and quality of the data sample. The correction mechanism proposed in this document has taken into account this fact.
2. Going into a deeper, microscopic analysis (e.g. a higher level of granularity or a more complex correction function) with the same sample data will not necessarily lead to a higher accuracy in the calculation of the total emissions, as the sample data may be capable of characterizing the traffic in Europe at a macroscopic level, but may not be capable of allowing a deeper analysis of microscopic phenomena.

### 3. PROPOSED RECONCILING METHODOLOGY

The reconciling methodology is based on the analysis of the fuel consumption data provided by the AOs. The methodology consists of the following steps:

#### 1 Adjustment of the fuel consumption coefficients

- 1.1 Calibration sample representativeness check and removal of the insufficient or non-representative sample data.
- 1.2 Trajectory correction factor (for flights where there is no distance in the CFMU database).
- 1.3 Calculation of fuel consumption coefficients for aircraft types included in the sample.
- 1.4 Assimilation of the rest of aircraft types.

#### 2 Recalculation of CO<sub>2</sub> historical emissions

The following diagram depicts the overall process of the reconciling methodology.

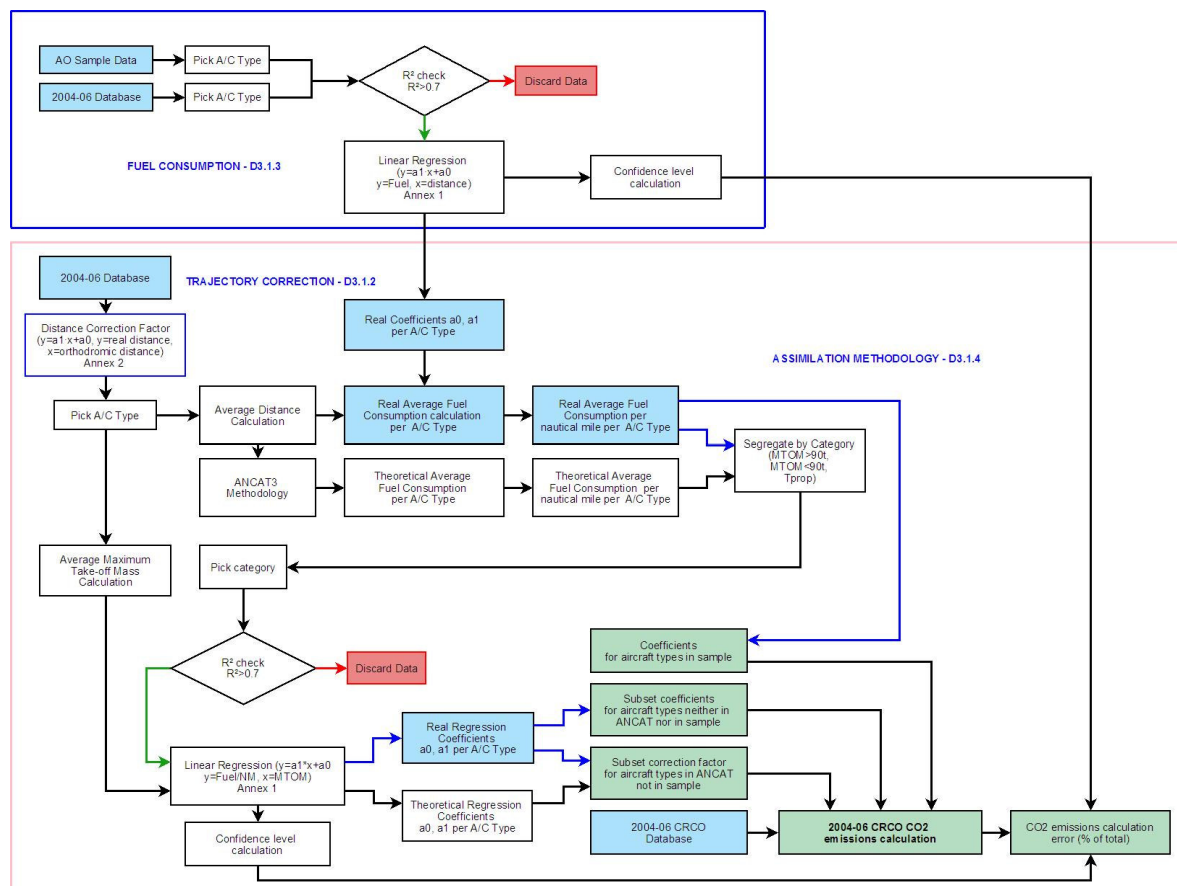


Figure 2. Proposed reconciling methodology – Flow diagram

### 3.1 Adjustment of the fuel consumption coefficients

#### 3.1.1 Verification of the representativeness of the calibration sample and removal of the insufficient and non-representative sample data

As it has been explained in section 2, there exist different factors not accounted for in the ANCAT-3 methodology that have an influence on fuel consumption (e.g. inefficiencies in the performance of the flights that cause delays during flight or on the ground). The sample data provided by the AOs corresponds to real flights and therefore account for these factors.

The sample data shall be statistically analysed to check its representativeness, in order to verify that the conclusions of the calibration process can be extrapolated to the whole set of flights included in the database. This depends on the amount of data provided by the AOs, but also on their quality.

The data included in the sample for each specific aircraft type shall be compared to the theoretical data in several dimensions. These dimensions include Maximum Take-Off Mass, route length, origin/destination airports... This is necessary in order to identify and correct any trend or gaps in the distribution of the sample.

Regarding route length, the average of the distances flown for the sample data should ideally be as close as possible to the average distance of the total population of flights. Given the sample data available at the moment of writing the present document and the limited margin to specify the required sample data or request additional data, this may not always be the case, which will result in a higher error and will be properly taken into account when computing the confidence interval.

#### 3.1.2 Trajectory correction

For flights for which there is no information on distance (from the CFMU database or from the information provided by National Authorities, as applicable), it is necessary to estimate the flown distance.

The proposed solution is to study the relationship between orthodromic distance and flight plan distance, which is done by means of a linear regression of known distances of flights and their corresponding orthodromic distance.

The details of such calculation are presented in Annex II.

#### 3.1.3 Correction factors for aircraft types included in the retained sample

For those aircraft types for which the sample has passed the representativeness check defined in 3.1.1, an analysis of the fuel consumption per aircraft type as a function of the flight distance shall be carried out.

Fuel consumption coefficients for a given aircraft type shall be calculated by computing the regression line for the sample following the method described in Annex I, so that:

$$y^{real} = \beta_0^{real} + \beta_1^{real} x \quad (1)$$

where  $y$  is the fuel consumption and  $x$  is the distance flown.

For each aircraft type regression, the **correlation coefficient** is calculated as per the formulas included in Annex I.



$R^2$  measures the strength of the linear relationship between  $x$  and  $y$ . Aircraft samples for which  $R^2 < 0.70$  are discarded.

For the retained aircraft types (valid samples), equation (1) will be used to estimate fuel consumption on a flight by flight basis.

In the function above, the coefficients  $\beta_0$  and  $\beta_1$  account in an aggregated manner for the different factors influencing the fuel consumption (meteorological conditions, airline strategy, taxiing, holding...).

### 3.1.4 Assimilation of the rest of aircraft types

Relying on the results from the previous section, the purpose of this part of the methodology is to define a set of formulas that, by entering an undefined aircraft type's Maximum Take Off Mass, will result in a fuel coefficient that will be multiplied by the distance travelled in order to compute the total fuel consumed, hence CO<sub>2</sub> emissions.

In order to complete this task, the following steps are proposed:

1. For each aircraft type of the valid sample<sup>1</sup>, the average distance flown and the average Maximum Take Off Mass are calculated for the 2004-2006 period.
2. Using the formula obtained in section 3.1.3, the fuel consumption is calculated for the average distance travelled by each aircraft type of the sample.
3. For each aircraft type, the result of (2) is divided by the average distance travelled, in order to acquire the *real average fuel consumption per nautical mile*.
4. Using ANCAT-3 Methodology, the fuel consumption is calculated for the average distance travelled by each aircraft type of the sample.
5. For each aircraft type, the result of (4) is divided by the average distance travelled, in order to acquire the *theoretical average fuel consumption per nautical mile*.
6. The aircraft types are segregated into three subsets:
  - jets over 90 tons,
  - jets below 90 tons, and
  - turboprops.

The rationale for this segmentation is based on the methodology developed by EUROCONTROL and explained in ref. [6], and relies on the empiric observation that, for each of these three subcategories, fuel consumption per NM shows a strong linear dependence with MTOM.

7. Two linear regressions following Annex I methodology are done for each subset. The first is the regression of the average MTOM and the result of (3), this regression shall be referred to as *real regression*. The second regression is that of the average MTOM and the result of (5), this regression shall be referred to as *theoretical regression*.
8. The *real regression* and the *theoretical regression* are analysed against each other, to find the correction factor of each subset.
9. A confidence interval for the *real regression* is determined according to equation [16] of Annex I.

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<sup>1</sup> Only those aircraft types for which the sub-sample has been considered valid according to the criteria defined in section 3.1.1 and 3.1.3 are retained.

10. Depending on the aircraft type, the coefficients to be used for the calculation of fuel consumption are selected as follows:

- The data of the sample are considered to be more exact than ANCAT; therefore, the coefficients to be used when an aircraft is present on the sample are those calculated as per section 3.1.3.
- The assimilated aircraft in ANCAT will maintain the assimilation factor, unless both, the primary and the assimilated, are in the sample, in which case, the coefficients calculated as per section 3.1.3 are applied to both types.
- For those aircraft not included in the sample, but defined by ANCAT, the ANCAT fuel consumption coefficients are corrected (shifted) by the correction factor acquired in (8), i.e. by the ratio between *real regression* and the *theoretical regression* for that MTOM value.
- For those aircraft not included in the sample, but belonging to the same family of one of the aircraft types included in the sample, the fuel consumption coefficients of the aircraft included in the sample are used, with a correction obtained from the ratio between the MTOM of the two models.
- For the rest of aircraft neither in ANCAT nor in the sample, the *real regression* equation is used to calculate the fuel consumption coefficient.

### 3.2 Recalculation of historical CO<sub>2</sub> aviation emissions

The output of the previous section is a new set of fuel consumption coefficients that account for the additional fuel consumption influencing factors described in section 2, as well as a correction factor for unknown flight plan distance between aerodromes.

These new factors need to be implemented into the methodology developed by EUROCONTROL and described in ref. [3] in the following manner:

1. **Acquisition of Data.**

No change is necessary on this section.

2. **Calculation of actual route length.**

For those flights where the flight plan route is unknown, the Trajectory correction factor needs to be implemented following the methodology described in section 3.1.2.

3. **Calculation of emissions on a flight-by-flight basis.**

Depending on the type of aircraft, the fuel consumption coefficients acquired as per section 3.1.3 or section 3.1.4 (as applicable) shall be implemented. The **confidence interval** for each individual flight shall be calculated for a given confidence level according to the instructions provided in Annex I, by applying equation [16].

4. **Directive exemptions.**

It will be necessary to recalculate the emissions of the directive exemptions, in case they are used for gap estimation.

5. **Gap emissions estimation.**

Gap emissions estimation shall be recalculated based on the new fuel consumption coefficients.

**6. De minimis Exemption Filtering.**

This section needs to be recalculated since the emissions estimation will have changed.

**7. Total CO<sub>2</sub> historical emissions.**

Recalculation is needed to compute final estimation of total CO<sub>2</sub> historical emissions.

**8. Calculation of global confidence interval.**

The confidence level of the resulting value for total CO<sub>2</sub> historical emissions shall be calculated by using the expression for error propagation provided in Annex III.

## 4. SUMMARY AND RECOMMENDATIONS

The reconciling methodology accomplishes the purpose of identifying improvement opportunities in the methodology used by EUROCONTROL.

The main approach taken to achieve these improvements is a macro-scale correction of the fuel consumption coefficients of the ANCAT 3 Methodology used by EUROCONTROL. The proposed statistical approach relies on the representativeness of the fuel consumption sample submitted by the AOs. The result is a set of correction factors depending on the type of aircraft.

The second correction, of much less impact, is the actual distance estimation for those flights for which there is not enough information in the flight plan.

The reconciling factors do not vary the fundamental principles of the initial methodology, but will however allow to improve the accuracy of the assessment thus reducing the risk of underestimating or overestimating the total emissions for 2004-2006.

## ANNEX I. LINEAR REGRESSION

The objective of this type of study is to adjust the parameters of a model function so as to best fit a data set. In our case, a linear regression model is proposed.

Given the collection of points  $(x_i, y_i)$ , where  $x_i$  is the flight distance and  $y_i$  is the fuel consumption, for  $i = 1, \dots, n$ , being  $n$  the total number of flights performed with a specific aircraft type included in the sample, we intend to determine the regression line that best fits the collection points. This regression line is as follows:

$$y = \beta_0 + \beta_1 x \quad (1)$$

### Correlation coefficient

Prior to obtaining the regression coefficients, we need to confirm the existence of a linear dependence between  $x$  and  $y$ . For this purpose, we obtain the linear Pearson correlation coefficient  $r$ , whose expression is:

$$r = \frac{b_{xy}}{b_x b_y} \quad (2)$$

where the moments to the centre  $b_x$  and  $b_y$  are calculated as follows:

$$b_{xy} = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) \quad (3)$$

$$b_x^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \quad (4)$$

This coefficient is  $-1 \leq r \leq 1$ .

The correlation is 1 in the case of an increasing linear relationship, -1 in the case of a decreasing linear relationship, and some value in between in all other cases, indicating the degree of linear dependence between the variables. The closer  $r^2$  is to 1, the stronger the linear relationship between the variables, therefore  $r^2$  provides an indication of the error between the variable  $y$  and its regression (for example, if  $r^2$  is 0.95, then 95% of the variance of  $y$  can be explained by changes in  $x$ ):

- if  $r^2$  is close to 1, it indicates that the variable  $y$  has a strong linear dependence on  $x$  and the error between the variable  $y$  and its regression is low.
- if  $r^2$  is close to 0, the variables are not related and the error between the variable  $y$  and its regression is high.

For the purpose of our study, those aircraft samples for which  $r^2 < 0,7$  are discarded.

## Calculation of linear regression coefficients

Using the Least Squares Method, we need to find the  $\beta_1$  and  $\beta_0$  to that minimize the following sum of squares (SS):

$$SS(\beta_0, \beta_1) = \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_i)^2 \quad (5)$$

The coefficients that minimize SS are calculated by using the following equations:

$$\beta_0 = \bar{y} - \beta_1 \bar{x} \quad (6)$$

$$\beta_1 = \frac{b_{xy}}{b_x^2} \quad (7)$$

## Calculation of confidence interval

Finally, we compute the confidence intervals on parameters:

- Confidence interval for  $\beta_1$

$$CI(\beta_1) = \beta_1 \pm t_{n-2, e/2} S_{\beta_1} \quad (8)$$

- Confidence interval for  $\beta_0$

$$CI(\beta_0) = \beta_0 \pm t_{n-2, e/2} S_{\beta_0} \quad (9)$$

where the standard deviation of the coefficients are

$$S_{\beta_0} = S \sqrt{\left( \frac{1}{n} + \frac{\bar{x}^2}{S_{xx}} \right)} \quad (10)$$

$$S_{\beta_1} = \frac{S}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}} \quad (11)$$

To compute these coefficients we need:

$$S = \sqrt{\frac{\sum_{i=1}^n (y_i - y(x_i))^2}{n-2}}, \quad S_{xx} = \sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i\right)^2}{n} \quad (12)$$

where  $y(x_i)$  is the regression evaluated at  $x_i$  distance.

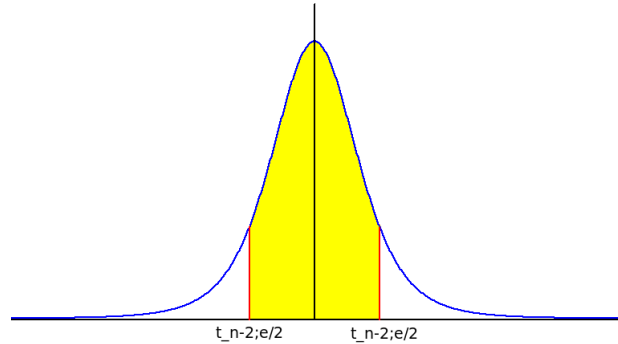
It can be proved that:

$$\frac{\beta_0}{S_{\beta_0}} \rightarrow t_{n-2, e/2}, \quad \frac{\beta_1}{S_{\beta_1}} \rightarrow t_{n-2, e/2} \quad (13)$$

i.e., the above variables follow a Student's T distribution with n-2 degrees of freedom.

$t_{n-2; e/2}$  is determined by the relationship

$$P(|T| < t_{n-2; e/2}) = 1 - e \quad (14)$$



The Confidence Interval estimate for the mean of y (lineal regression) given a particular x is as follows:

$$\beta_0 + \beta_1 x \pm t_{n-2; e/2} S \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{S_{xx}}} \quad (15)$$

The Confidence Interval estimate for an individual value of y given a particular x is as follows:

$$\beta_0 + \beta_1 x \pm t_{n-2; e/2} S \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{S_{xx}}} \quad (16)$$

## ANNEX II. DISTANCE CORRECTION

Using the Least Squares Method, we need to find the coefficients  $\delta_0$  and  $\delta_1$  that minimize the following sum of squares:

$$SS(\delta_0, \delta_1) = \sum_{i=1}^n (x_i^{real} - \delta_0 - \delta_1 x_i^{orth})^2 \quad (1)$$

and using the method described in Annex I, we obtain the correction function:

$$x^{real} = CF(x^{orth}) = \delta_0 + \delta_1 x^{orth} \quad (2)$$

This function describes the relationship between the orthodromic distance and the flight plan distance, and allows to estimate the actual route length  $x^{real}$  as a function  $CF(x^{orth})$  for flights whose actual route length is not available.



## ANNEX III. ERROR PROPAGATION

**Propagation of error** (or **propagation of uncertainty**) is the effect of variables' uncertainties or errors on the uncertainty of a function based on them.

In our case, we need to estimate several quantities (corresponding to the emissions of different aircraft types) and then add them together to get the final result. Error propagation is used to combine the errors due to the different estimations and get the confidence interval of the total estimated historical emissions.

Being  $q$  the total emissions calculated from the addition of the emissions of different subsets of flights  $X_i$  (i.e.  $X_T = X_1 + X_2 + \dots + X_n$ ), where each  $X_i$  corresponds for example to the emissions due to a certain aircraft type and has an associated confidence interval  $\delta_i$ , we need to calculate the absolute uncertainty for  $q$ .

$$q = f(x_1, x_2, \dots, x_n) \quad (1)$$

As the errors are independent and random in each variable, then the error is given by:

$$\delta q = \sqrt{\left(\frac{\partial f}{\partial x_1} \delta x_1\right)^2 + \left(\frac{\partial f}{\partial x_2} \delta x_2\right)^2 + \dots + \left(\frac{\partial f}{\partial x_n} \delta x_n\right)^2} \quad (2)$$

As we have:

$$q = f(x_1, x_2, \dots, x_n) = x_1 + x_2 + \dots + x_n \quad (3)$$

The total error is obtained by adding the absolute uncertainties in quadrature:

$$\delta q = \sqrt{(\delta x_1)^2 + (\delta x_2)^2 + \dots + (\delta x_n)^2} \quad (4)$$