

# Demographics and Real Interest Rates Across Countries and Over Time\*

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**The monetary and fiscal policy mix in a changing world  
European Commission**

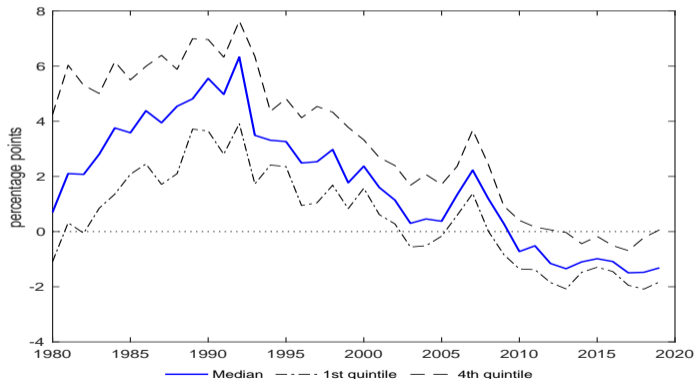
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# Real Interest Rates in Advanced Economies

- Between 1990 and 2019, real interest rates exhibited
  1. Large persistent decline
  2. Falling dispersion across countries



Note: Ex-ante real short-term interest rates (nominal yields minus expected inflation) for 19 OECD countries

# Real Interest Rates in Advanced Economies

- Between 1990 and 2019, real interest rates exhibited
  1. Large persistent decline
  2. Falling dispersion across countries
- Demographic trends among candidate explanations for secular decline of real interest rates
  - ▶ Lots of existing work in closed economy, including [Carvalho, Ferrero and Nechio \(2016\)](#)
- **How do demographic trends affect real interest rates in open economy?**
  - ▶ Domestic vs. foreign demographics
  - ▶ Interaction of demographics and (imperfect) capital mobility

# What We Do and What We Find

1. Calibrate and simulate **multicountry life-cycle model with imperfect capital mobility**
  - ▶ Demographic trends account for about  $1/3$  of decline in real rates between 1990 and 2019
  - ▶ Financial integration shifts sensitivity of real interest rate towards global demographics
  - ▶ Can explain about  $1/2$  of falling dispersion across countries

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  - ▶ Can explain about  $1/2$  of falling dispersion across countries
2. Estimate **panel ECM** on data between 1980 and 2019 accounting for financial integration
  - ▶ Global factors significant across specifications
  - ▶ Domestic demographics significant after controlling for fiscal policy
  - ▶ Among other domestic variables, pension spending emerges as most robust driver

# Related Literature

## ● Quantitative models of demographics and real interest rates

- ▶ Krueger and Ludwig (2006), Ikeda and Saito (2014), Kara and Von Thadden (2016), Carvalho, Ferrero and Nechio (2016), Eggertsson, Merhotra and Robbins (2017), Sudo and Takizuka (2018), Bielecki, Brzoza-Brzezina and Kolasa (2020), Gagnon, Johansen and Lopez-Salido (2021), Lisack, Sajedi, and Thwaites (2021), Auclert, Malmberg, Martenet and Rognlie (2021), Kopecky and Taylor (2022), Sposi (2022),...

## ● Empirical analysis of real interest rates dynamics with focus on demographics

- ▶ Rachel and Smith (2015), Favero, Gozluklu and Yang (2016), Hamilton, Harris, Hatzius and West (2016), Yi and Zhang (2016), Fiorentini, Galesi, Perez-Quirós and Sentana (2018), Aksoy, Basso, Smith and Grasl (2019), Lunsford and West (2019), Borio, Disyatat, Juselius and Rungcharoenkitkul (2021), Davis, Fuenzalida, Huetsch, Mills and Taylor (2024),...

## ● Other determinants of low/declining real interest rates

- ▶ Caballero, Farhi and Gourinchas (2008), Gomme, Ravikumar and Rupert (2015), Sajedi and Thwaites (2016), Caballero and Farhi (2017), Del Negro, Giannone, Giannoni and Tambalotti (2017, 2018), Holston, Laubach and Williams (2017), Farhi and Gorio (2018), Rachel and Summers (2019), Eggertsson, Robbins and Wold (2021), Ferreira and Shousha (2021), Reis (2022), Schmitt-Grohé and Uribe (2022), Obstfeld (2024), ...

# Outline

- Introduction
- **Open-economy life-cycle model**
- Panel ECM

# Model Overview

- Open-economy life-cycle model with imperfect capital mobility
  - ▶ Life-cycle structure as in [Gertler \(1999\)](#)
  - ▶ Time-varying demographics, heterogeneous across countries ([Ferrero, 2010](#))
  - ▶ Portfolio-holding costs hamper international capital mobility ([Chang, Liu and Spiegel, 2015](#))
- Continuum of workers ( $w$ ) and retirees ( $r$ ) in each country  $m \in \{1, \dots, \mathcal{M}\}$ 
  - ▶ Face idiosyncratic risk of retirement (for workers) and death (for retirees)
  - ▶ Consume one good and can save through capital, government bonds or claims on foreign assets
- Standard supply side (labor-augmenting productivity)
- Government funds spending and pensions with lump-sum taxes and debt



# Demographics

- Life-cycle structure in country  $m$ 
  - ▶ Each period  $(1 - \omega_{mt} + n_{mt})N_{mt-1}^w$  new individuals are born workers
  - ▶ Workers remain in labor force with probability  $\omega_{mt}$ , retire otherwise
  - ▶ Retirees survive with probability  $\gamma_{mt}$

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$$N_{mt}^w = (1 + n_{mt})N_{mt-1}^w$$

- Old dependency ratio

$$\psi_{mt} \equiv \frac{N_{mt}^r}{N_{mt}^w} = \frac{(1 - \omega_{mt}) + \gamma_{mt}\psi_{mt-1}}{1 + n_{mt}}$$

# Retirees' Problem

- Retiree born in period  $j$  and retired in period  $k$  solves

$$V_{mt}^{rjk} = \max_{C_{mt}^{rjk}, \{A_{m\ell t}^{rjk}\}_{\ell=1}^{\mathcal{M}}} \left[ \left( C_{mt}^{rjk} \right)^{\frac{\sigma-1}{\sigma}} + \beta_m \gamma_{mt+1} \left( V_{mt+1}^{rjk} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

subject to

$$C_{mt}^{rjk} + \left[ 1 + \sum_{\ell \neq m}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left( \eta_{m\ell t}^{rjk} \right)^2 \right] \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}^{rjk} = \frac{1}{\gamma_{mt}} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{rjk} + E_{mt}^{rjk}$$

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- $\Lambda_{m\ell t} \rightarrow$  Portfolio-holding cost (stand-in for frictions that hamper international capital mobility)
- $\eta_{m\ell t}^{rjk} \equiv A_{m\ell t}^{rjk} / (\sum_{p=1}^{\mathcal{M}} A_{mp t}^{rjk}) \rightarrow$  Portfolio share of country  $m$  vis-à-vis country  $\ell$

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- Retirees discount future taking into account probability of surviving
  - Turn their wealth to competitive domestic portfolio managers that pool death risk (Yaari, 1965; Blanchard, 1986)

# Workers' Problem

- Worker born in period  $j$  solves

$$V_{mt}^{wj} = \max_{C_{mt}^{wj}, \{A_{m\ell t}^{wj}\}_{\ell=1}^{\mathcal{M}}} \left\{ \left( C_{mt}^{wj} \right)^{\frac{\sigma-1}{\sigma}} + \beta_m \left[ \omega_{mt+1} V_{mt+1}^{wj} + (1 - \omega_{mt+1}) V_{mt+1}^{rjt+1} \right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}$$

subject to

$$C_{mt}^{wj} + \left[ 1 + \sum_{\ell \neq m}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left( \eta_{m\ell t}^{wj} \right)^2 \right] \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}^{wj} = \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{wj} + W_{mt}^w - T_{mt}^w$$

- ▶ No initial assets
- ▶ Risk-neutral with respect to labor income risk
- ▶ Uninsurable retirement risk



# Portfolios and Wealth Distribution

- Retirees' portfolio share in country  $p \neq m$  independent of age and time since retirement

$$\left[ 1 + \sum_{\ell \neq m}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left( \eta_{m\ell t}^{rjk} \right)^2 \right] (R_{pt} - R_{mt}) = \Lambda_{mpt} \eta_{mpt}^{rjk} R_{mt} \Rightarrow \eta_{mpt}^{rjk} = \eta_{mpt}^r$$

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- Portfolio costs prevent full return equalization across countries

- Same condition holds for workers ( $\eta_{mpt}^{wj} = \eta_{mpt}^w$ )  $\Rightarrow \eta_{mpt}^r = \eta_{mpt}^w$

- Country- $m$  retirees' asset share vis-à-vis country  $p$  equal to their overall wealth share**

$$\lambda_{mpt} \equiv \frac{A_{mpt}^r}{A_{mpt}^r + A_{mpt}^w} = \frac{A_{mt}^r}{A_{mt}^r + A_{mt}^w} = \lambda_{mt}$$

- Evolution of wealth easy to track

# Aggregate Consumption

- Marginal propensity to consume independent of individual characteristics

- ▶ Retirees

$$(\bar{\zeta}_{mt}^r)^{-1} = 1 + \gamma_{mt+1} \beta_m^\sigma \tilde{R}_{mt}^{\sigma-1} (\bar{\zeta}_{mt+1}^r)^{-1}$$

- ▶ Workers

$$(\bar{\zeta}_{mt}^w)^{-1} = 1 + \beta_m^\sigma (\Omega_{mt+1} \tilde{R}_{mt})^{\sigma-1} (\bar{\zeta}_{mt+1}^w)^{-1}$$

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- Easy to aggregate consumption within each group

- ▶ Retirees

$$C_{mt}^r = \bar{\zeta}_{mt}^r \left( \frac{1}{\gamma_{mt}} \sum_{\ell=1}^M R_{\ell t-1} A_{m\ell t-1}^r + \underbrace{S_{mt}^k}_{\text{PDV of pension benefits to } r} \right)$$

- ▶ Workers

$$C_{mt}^w = \bar{\zeta}_{mt}^w \left( \sum_{\ell=1}^M R_{\ell t-1} A_{m\ell t-1}^w + \underbrace{H_{mt}^w}_{\text{PDV of human wealth}} + \underbrace{Z_{mt}^w}_{\text{PDV of pension benefits to } w} \right)$$

[details](#)

# Production

- Perfectly competitive **firms** produce homogeneous consumption good
  - ▶ Labor-augmenting Cobb-Douglas technology

$$Y_{mt} = (X_{mt} N_{mt}^w)^\alpha K_{mt-1}^{1-\alpha} \quad \text{where} \quad X_{mt} = (1 + x_{mt}) X_{mt-1}$$

- ▶ Law of motion of capital

$$K_{mt} = (1 - \delta) K_{mt-1} + I_{mt}$$

# Production and Government

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- ▶ Law of motion of capital

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- **Government** budget constraint

$$G_{mt} + E_{mt} + R_{mt-1} B_{mt-1} = T_{mt} + Y_{mt} + B_{mt}$$

- ▶ Spending, pensions, and debt are exogenous shares of GDP

$$G_{mt} = g_{mt} Y_{mt} \quad E_{mt} = e_{mt} Y_{mt} \quad B_{mt} = b_{mt} Y_{mt}$$



# Balance of Payments

- Country- $m$  assets held by residents

$$A_{mmt} = K_{mt} + B_{mt} - \sum_{\ell \neq m} A_{\ell mt}$$

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$$\sum_{\ell \neq m} (A_{m\ell t} - A_{\ell mt}) \equiv F_{mt}$$

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- Global asset market clearing**

$$\sum_{\ell} F_{\ell t} = 0$$

# Initial Steady State

- **Three countries** with different demographic profiles

	<b>"Young" (<math>\mathcal{Y}</math>)</b>	<b>"Old" (<math>\mathcal{O}</math>)</b>	<b>ROW (<math>\mathcal{W}</math>)</b>
Relative size ( $N_{m0}^w / N_{\mathcal{W}0}^w$ )	0.007	0.007	1
Growth rate of labor force ( $n_{m0}$ )	1.13	0.55	0.76
Dependency ratio ( $\psi_{m0}$ )	20.48	25.08	22.34

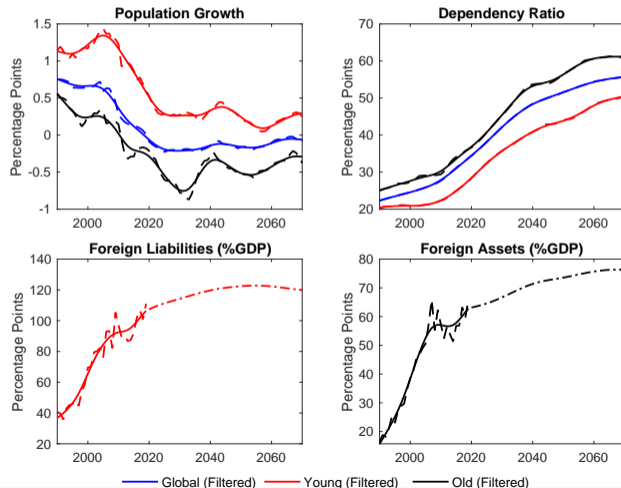
- Additionally set country-specific  $\beta$  and  $\Lambda_{1990}$  to match distribution of  $R_{1990}$  and  $F_{1990}$

	<b>"Young" (<math>\mathcal{Y}</math>)</b>	<b>"Old" (<math>\mathcal{O}</math>)</b>	<b>ROW (<math>\mathcal{W}</math>)</b>
$R_{1990}$	6.76	2.37	5.12
$F_{1990}$	-36.76	15.73	0

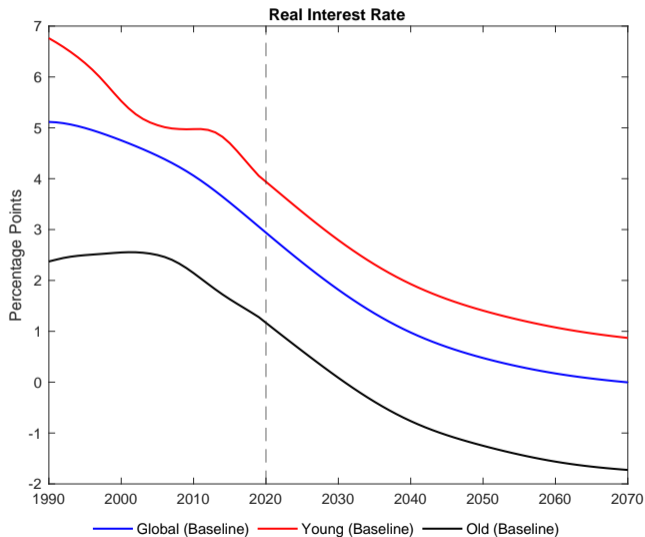
- All other parameters common across countries ([Carvalho, Ferrero and Nechio, 2016](#)) details
  - ▶ Period is one year
  - ▶ Workers born at age 20, retire on average at 65

# Calibrating Transition

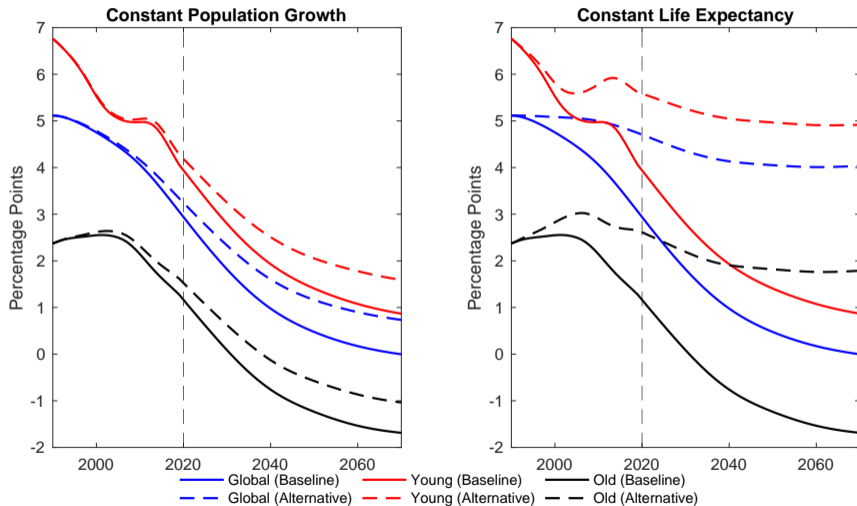
- Feed model with data and projections for **demographics** from 1990 to 2070 (UN WPD 2019)
- Back out **portfolio costs** that match HP-filtered foreign asset positions (Lane and Milesi-Ferretti, 2017)



# Transition Results

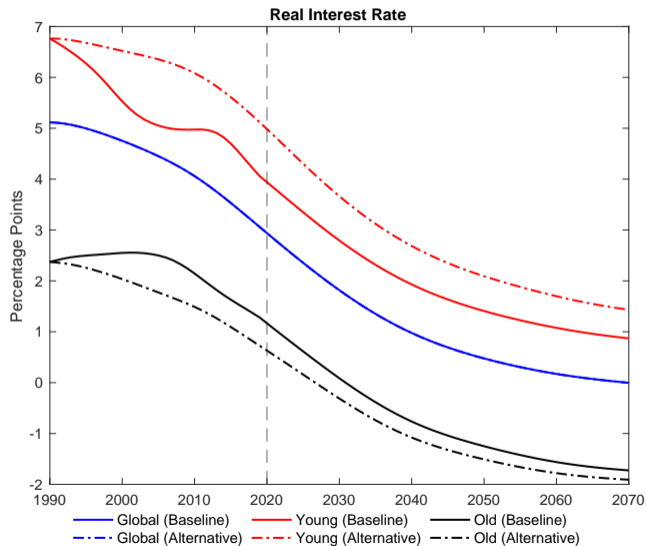


# Demographics Counterfactuals





# Financial Integration Counterfactual



## Other Factors

- Focus on demographics ([more details](#)) but model also features other factors relevant for real rates
- Steady state comparative statics exercise (for small old economy)

Factor	Baseline	Alternative	$\Delta R^O$ (in bps)	
			1990	2020
TFP ( $x$ )	0.5%	0.6%	17	11
Debt / GDP ( $b$ )	60%	70%	14	8
Government spending / GDP ( $g$ )	25%	26%	20	13
Pensions / GDP ( $e$ )	7.5%	8.5%	56	36
Retirement age ( $\omega$ )	65	66	64	31

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- **Pensions versus government spending: incentives to save.**

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- Open-economy life-cycle model
- **Panel ECM**

# Empirical Approach and Data

- Perform empirical analysis informed by model predictions
  - ▶ **Regress real interest rates on demographic variables controlling for financial integration**
  - ▶ Also control for other factors following existing literature
    - ★ TFP growth, government debt, pension spending, convenience yields, inequality

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- Data at annual frequency (1979–2019) from various sources (World Bank, IMF, OECD, UN)
- Unbalanced panel of 19 OECD countries
  - ▶ Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, United Kingdom, and United States

# Empirical Specification

- **Error Correction Model (ECM)** to focus on low-frequency movements

$$\begin{aligned} \Delta r_{m,t} = & \alpha_m + \gamma r_{m,t-1} + \theta \Theta_{m,t-1} r_{m,t-1}^* + \sum_j \psi_j (1 - \Theta_{m,t-1}) D_{m,j,t-1} + \sum_k \Psi_k (1 - \Theta_{m,t-1}) X_{m,k,t-1} \\ & + \lambda \Delta(\Theta_{m,t} r_{m,t}^*) + \sum_j \phi_j \Delta[(1 - \Theta_{m,t}) D_{m,j,t}] + \sum_k \chi_k \Delta[(1 - \Theta_{m,t}) X_{m,k,t}] + \epsilon_{m,t}, \end{aligned}$$

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- ▶  $r_{m,t}$  → Short-term rate minus one-year-ahead expected inflation ([Hamilton et al., 2016](#))



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- ▶  $D_{m,j,t}$  → Demographic variables

# Empirical Specification

- Error Correction Model (ECM) to focus on low-frequency movements

$$\begin{aligned} \Delta r_{m,t} = & \alpha_m + \gamma r_{m,t-1} + \theta \Theta_{m,t-1} r_{m,t-1}^* + \sum_j \psi_j (1 - \Theta_{m,t-1}) D_{m,j,t-1} + \sum_k \Psi_k (1 - \Theta_{m,t-1}) X_{m,k,t-1} \\ & + \lambda \Delta(\Theta_{m,t} r_{m,t}^*) + \sum_j \phi_j \Delta[(1 - \Theta_{m,t}) D_{m,j,t}] + \sum_k \chi_k \Delta[(1 - \Theta_{m,t}) X_{m,k,t}] + \epsilon_{m,t}, \end{aligned}$$

- ▶  $r_{m,t}$  → Short-term rate minus one-year-ahead expected inflation ([Hamilton et al., 2016](#))
- ▶  $D_{m,j,t}$  → Demographic variables
- ▶  $X_{m,k,t}$  → Other controls

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- ▶ For each country  $m$ , **world real rate summarizes global factors**

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- ▶ Interact domestic variables with complement of measure of **financial integration**

$$\Theta_{m,t} = \frac{LMF_{m,t}}{100 + LMF_{m,t}}$$

where  $LMF_{m,t}$  is sum of financial assets and liabilities/GDP (Lane and Milesi-Ferretti, 2017)

# ECM Results ( $\hat{\beta}/\hat{\gamma}$ )

	(1)	(2)	(3)	(4)	(5)	(6)
Global Rate	0.68*** (0.17)	0.66*** (0.17)	0.70*** (0.13)	0.74*** (0.14)	1.00*** (0.20)	1.48*** (0.20)
Life Expectancy	0.14*** (0.04)	0.14*** (0.04)	-0.24*** (0.06)	-0.17 (0.19)	-0.36*** (0.09)	-0.54* (0.28)
Growth Rate of Labor Force	0.24 (1.02)	0.30 (1.01)	6.03*** (0.98)	6.12*** (1.10)	8.95*** (1.51)	11.59*** (1.49)
TFP Growth		0.49 (0.34)	0.02 (0.30)	-0.14 (0.37)	-0.02 (0.39)	-0.01 (0.41)
Government Debt			0.03 (0.02)	0.01 (0.03)	0.07** (0.03)	0.10** (0.04)
Pension Spending			2.31*** (0.41)	2.11*** (0.59)	2.12*** (0.53)	2.65*** (0.80)
Gini Coefficient				-0.05 (0.23)		-0.03 (0.33)
Convenience Yield					0.67 (1.35)	1.99 (1.68)
Lagged real rate	-0.31*** (0.03)	-0.32*** (0.03)	-0.46*** (0.03)	-0.50*** (0.04)	-0.53*** (0.06)	-0.68*** (0.06)
Kao test	R***	R***	R***	R***	R***	R***
$R^2$	0.24	0.24	0.39	0.36	0.53	0.55
Adjusted $R^2$	0.21	0.22	0.35	0.31	0.48	0.48
Observations	743	743	505	445	206	169
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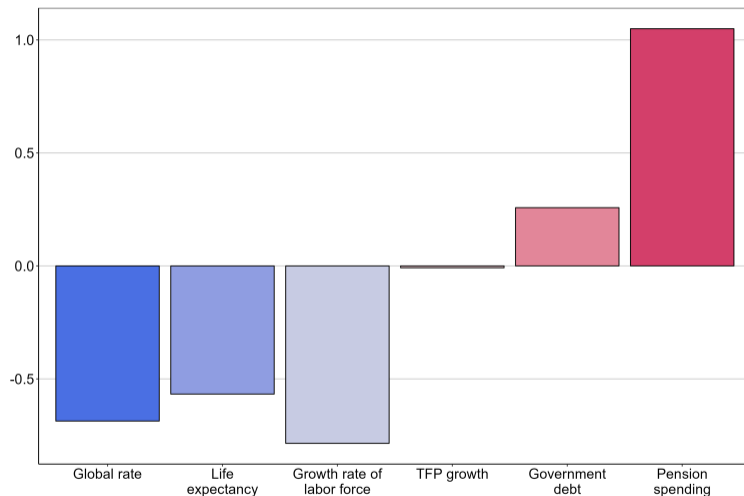
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# Interpreting the Coefficients



- **Estimates for pensions and debt in line with comparative statics.**

1990-2019

# No Financial Integration ( $\hat{\beta} / \hat{\gamma}$ )

	(1)	(2)	(3)	(4)	(5)	(6)
Global Rate	0.32** (0.13)	0.30** (0.13)	-0.23 (0.15)	-0.36*** (0.13)	-0.24 (0.21)	-0.21 (0.17)
Life Expectancy	-0.76*** (0.14)	-0.72*** (0.13)	-1.49*** (0.16)	-1.43*** (0.16)	-0.95*** (0.30)	-1.07*** (0.31)
Growth Rate of Labor Force	-0.58 (0.35)	-0.41 (0.36)	0.58 (0.41)	0.04 (0.36)	1.64** (0.74)	1.16* (0.63)
TFP Growth		0.31** (0.14)	0.24* (0.14)	0.05 (0.14)	0.34* (0.20)	0.16 (0.19)
Government Debt			0.01 (0.01)	-0.02** (0.01)	0.01 (0.01)	-0.02* (0.01)
Pension Spending			0.47** (0.21)	-0.03 (0.20)	0.71** (0.28)	0.05 (0.26)
Gini Coefficient				0.08 (0.09)		-0.00 (0.14)
Convenience Yield					-1.90** (0.85)	-0.53 (0.78)
Lagged real rate	-0.33*** (0.03)	-0.34*** (0.03)	-0.42*** (0.03)	-0.53*** (0.04)	-0.48*** (0.05)	-0.66*** (0.06)
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- **Empirics**
  - ▶ Global factors and domestic demographics with sizeable contributions for decline
  - ▶ Pension spending main factor pushing in opposite direction

# Appendix

# Portfolio Shares and Evolution of Wealth Distribution

- Same portfolio shares ( $\eta_{mpt}^r = \eta_{mpt}^w = \eta_{mpt}$ ) imply

$$\lambda_{mpt} \equiv \frac{A_{mpt}^r}{A_{mpt}^r + A_{mpt}^w} = \frac{\eta_{mpt} A_{mt}^r}{\eta_{mpt} A_{mt}^r + \eta_{mpt} A_{mt}^w} = \frac{A_{mt}^r}{A_{mt}^r + A_{mt}^w} = \lambda_{mt}$$

- Combining  $r$  and  $w$  budget constraints, financial wealth evolves according to

$$\begin{aligned} & \left[ 1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell}}{2} (\eta_{m\ell t} - \bar{\eta}_{m\ell})^2 \right] [\lambda_{mt} - (1 - \omega_{mt+1})] A_{mt} \\ & = \omega_{mt+1} \left[ (1 - \zeta_{mt}^r) \lambda_{mt-1} A_{mt-1} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} \eta_{m\ell t-1} + E_{mt} - \zeta_{mt}^r S_{mt} \right] \end{aligned}$$

# Drivers of Consumption

- Retirees' consumption depends on financial wealth and PDV of pension benefits

$$S_{mt}^{rjk} = E_{mt}^{rjk} + \frac{\gamma_{mt+1} S_{mt+1}^{rjk}}{\tilde{R}_{mt}}$$

where  $\tilde{R}_{mt}$  is total adjusted return

$$\tilde{R}_{mt} \equiv \frac{\sum_{\ell=1}^{\mathcal{M}} \eta_{m\ell t} R_{\ell t}}{1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} (\eta_{m\ell t})^2}$$

- Workers' consumption depends on PDV of net labor income

$$H_{mt}^{wj} = W_{mt}^w - T_{mt}^w + \frac{\omega_{mt+1} H_{mt+1}^{wj}}{\Omega_{mt+1} \tilde{R}_{mt}}$$

and PDV of pension benefits

$$Z_{mt}^{wj} = \frac{1}{\Omega_{mt+1} \tilde{R}_{mt}} \left[ (1 - \omega_{mt+1}) \left( \frac{\bar{\zeta}_{mt+1}^r}{\bar{\zeta}_{mt+1}^w} \right)^{\frac{1}{1-\sigma}} S_{mt+1}^{rjt+1} + \omega_{mt+1} Z_{mt+1}^{wj} \right]$$

# Parameters

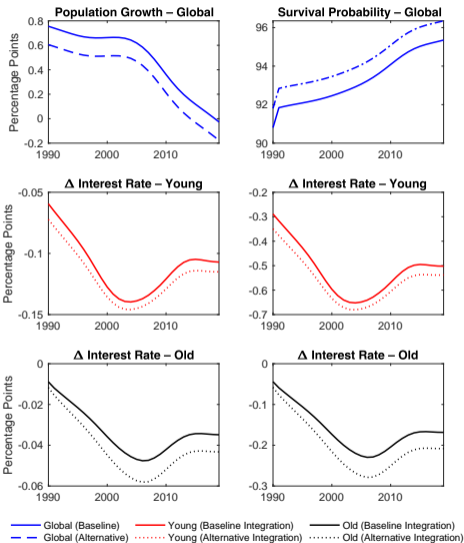
## Country-specific parameters

Parameter	$\mathcal{Y}$	$\mathcal{O}$	$\mathcal{W}$
Discount factor ( $\beta_m$ )	0.992	1.023	1.004
Portfolio-holding costs ( $\Lambda_{\mathcal{Y}n0}$ )	0	300	19.2
Portfolio-holding costs ( $\Lambda_{\mathcal{O}n0}$ )	300	0	64.6
Portfolio-holding costs ( $\Lambda_{\mathcal{W}n0}$ )	19.2	64.6	0

## Parameters common across countries

Parameter value	Description
$\omega = 0.978$	Average employment duration
$\sigma = 0.500$	Elasticity of intertemporal substitution
$\alpha = 0.667$	Labor share
$\delta = 0.100$	Depreciation rate
$x = 0.005$	Growth rate of productivity
$\bar{\eta} = 0$	Target net foreign asset position
$b = 0.600$	Debt/GDP
$g = 0.250$	Government spending/GDP
$e = 0.075$	Pensions/GDP

# Comparative Statics for Demographic Trends



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# Interpreting the Coefficients (Post-1990)

