The Costs and Beliefs Implied by Direct Stock Ownership

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Introduction

Household direct stock ownership has been documented in:

- U.S. and European survey data
- U.S. and European brokerage data
- European tax data
- And now, likely in the ECB data

Direct stock ownership motivated by investor beliefs

- Learning about skill through trading: Linnainmaa (2011), Seru et. al. (2010)
- Engelberg et. al. (2012): Jim Cramer stock picks
- Familiarity Bias: Massa and Siminov (2006), others
- Trading on news: Barber and Odean (2007)
- Over-confidence: many (see paper)

Introduction

Contribution:

1. Develop model of household research costs, beliefs, and direct stock ownership
2. Structurally estimate distribution of household beliefs and research costs
   - Intuition: Beliefs should be reflected in broad asset allocations, not just trading behavior
   - Compare to Linnainmaa (2011)
3. Identify structural parameters using only households' wealth and portfolio choices
   - Compare to Anderson (2013)
4. Show model matches a number of empirical facts about household portfolios

Data

- Survey of Consumer Finances (SCF): Use 1995, 1998, 2001, 2004 and 2007 waves
- SCF is triennial cross-sectional survey of U.S. households
- Data on all (almost all) financial assets: cash, checking accts, saving accts, bonds, stocks, mutual funds, retirement accts, etc.
- Define *Wealth* as *Total Financial Wealth*: all cash, investments and retirement accounts; exclude real estate, insurance, and debt/credit
- Possible to do something similar with HFCS data - although don't have info on number of stocks held

Data

- Inclusion criteria, drop:
  - wealth and age outliers
  - those with no equity (diversified or direct)
  - own-firm stockholders
  - non-active investors
- Results in 1,767 observations

Details on sample criteria

Summary Statistics

	mean	st. dev.	min	max
Age	44.0	10.6	22.0	64.0
Annual Income	$ 84,366.0	$ 113,210.8	$ 0	$ 4,452,959.0
Total F. Wealth	$ 260,388.5	$ 751,895.7	$ 1,010.0	$ 29,200,000.0
Married	67.0%	-	-	-
% w/ Stocks	19.4%	-	-	-
# of Stocks	8.3	12.5	1.0	150.0
# of Obs.	1,767	-	-	-

Stylized Facts from SCF

- Four main stylized facts:
  1. Likelihood of owning individual stocks increases wealth
  2. The expected number of individual stocks held increases wealth
  3. Fraction of total equity allocated to individual stocks increases with the number of individual stocks held
  4. Total equity share increases with the number of individual stocks held

Fact 1 - Likelihood of Holding Individual Stocks $\uparrow$ Wealth

Financial Wealth	# Obs.	% of Households w/ some Ind. Stocks
0-250k	1,018	13.6%
250k-500k	189	28.7%
500k-1M	162	43.8%
1M-2M	160	60.4%
2M-3M	61	59.1%
$>$ 3M	177	71.6%

*Robust to education, age, income, professional financial advice, and home ownership

Fact 2 - $\#$ Individual Stocks Held $\uparrow$ Wealth

Financial Wealth	**Median Number of Stocks Held**
0-250k	3
250k-500k	7
500k-1M	6
1M-2M	10
2M-3M	15
$>$ 3M	23

* of households with individual stocks.

Cannot only be about diversification Regression results Full table

Fact 3 / Fact 4 - More Stocks → Higher Allocations

Dep. Variable:	% of Equity in Ind. Stocks	% of Total Portfolio in Equity
# Ind. Stocks Held	0.014***	0.003***
	(0.002)	(0.001)
Fin. Advice	-0.024*	-0.025
	(0.014)	(0.018)
Education	0.007***	0.028***
	(0.002)	(0.002)
Age	-0.001	0.003***
	(0.001)	(0.001)
Observations	1,767	1,767
R-squared	0.283	0.788

*Income, Financial Wealth (and squared), OwnsHome included and insignificant.

Households

- Households (investors) denoted by i
- Have iso-elastic utility over consumption in each period
- In each period, households can research individual stocks
  - $\rightarrow$ learning information about stock's idiosyncratic return
  - But learning this information is costly
- Households are heterogeneous in research costs, beliefs, initial wealth
- All investors have same CRRA coefficient γ

Research Process

- In period t, household i may spend $\$q_{i,t}$ to learn about one stock *in expectation*
- Households only research stocks between ages of 22 and 64. "Retire" from active management at 65.
- Stochastic nature to research:
  - In each period t, household i chooses research level (intensity) $s_{i,t}$
  - $\rightarrow$ learns about $\hat{z}_{i,t}$ number of stocks: $\hat{z}_{i,t} \sim Poiss(s_{i,t})$
- Cost of research is $q_{i,t} \times s_{i,t}$: To learn $\tilde{z}$ alphas on average, must spend $q_{i,t} \times \tilde{z}$
- Research is $s_{i,t}$ not $\hat{z}_{i,t}$. Assume $s_{i,t}$ is integer valued
- Assume $\log(q_{i,t}) \sim N(\mu_q + \beta Y_{i,t}, \sigma_q^2)$; $Y_{i,t}$ vector of covariates

Assets and Returns

- Risk-free asset B
    - Gross risk-free return: $1 + R$
- Market (mutual) fund M
    - Stochastic gross log-return: $\log(1 + R_{M,t}) \sim N(\mu, \sigma^2)$
    - μ and σ^2 known

Assets and Returns

- N individual stocks $\{X_1, ..., X_N\}$
  - $\boldsymbol{1 + R_{j,t} = (1 + R_{M,t}) \times \varepsilon_{j,t} \times \alpha_{j,t}}$
  - $\varepsilon_{k,t}$ and $\alpha_{j,t}$ — mean-one, lognormal shocks:
  - $\varepsilon_{k,t}$ and $\alpha_{j,t}$ assumed independent of each other and $1 + R_{M,t}$
  - $\alpha_{j,t}$ — households believe is learnable (through research)
  - $\varepsilon_{j,t}$ — households believe is unlearnable
  - $\Rightarrow 1 + R_{j,t}$ is also lognormal

Household Beliefs

- $1 + R_{j,t} = (1 + R_{M,t}) \times \boldsymbol{\varepsilon}_{j,t} \times \boldsymbol{\alpha}_{j,t}$
- **If household i researches stock j in period t, believes to learn $\boldsymbol{\alpha_{j,t} = \hat{\alpha}_{i,j,t}}$**
  - Note: this is a deviation from rational expectations
- Household i believes:
  - $\boldsymbol{\log(\alpha_{j,t}) \sim N(0, \sigma^2_{\alpha,i})}$: $\sigma^2_{\alpha,i}$ is **predictable variance**
  - $\log(\varepsilon_{j,t}) \sim N(0, \sigma^2_{\varepsilon,i})$
- Beliefs about $\sigma^2_{\alpha,i} \Rightarrow$ beliefs about **fraction of non-market stock return variation that is predictable**
- Heterogeneity in beliefs $\rightarrow$ heterogeneity in $\sigma^2_{\alpha,i}$

Heterogeneous Beliefs

- To see this, define: $V = \mathrm{Var}(\log(1 + R_{j,t}))$.

  By construction: $V - \sigma^2 = \sigma^2_{\alpha,i} + \sigma^2_{\varepsilon,i}$

  Non-market (log) variance = unpredictable variance + predictable variance

- Assume fraction of log non-market variance that is predictable is distributed by a *Beta* distribution:
  - $\frac{\text{predictable variance}}{\text{non-market variance}} = \frac{\sigma^2_{\alpha,i}}{V - \sigma^2} \sim Beta(\phi, \tau)$

- Note: V, σ^2 come from data: $\sigma^2_{\alpha,i} \Rightarrow \sigma^2_{\varepsilon,i}$. No j or t subscripts on $\sigma^2_{\alpha,i}$ or $\sigma^2_{\varepsilon,i}$

A word on $1 + R_{j,t}$

$$E[\log(1+R_{j,t})] = \begin{cases} \mu + -\frac{1}{2}\sigma_\varepsilon^2 + \log(\hat{\alpha}_{i,j,t}) & \text{if } j \text{ researched} \\ \mu + -\frac{1}{2}\sigma_\varepsilon^2 + -\frac{1}{2}\sigma_\alpha^2 & \text{otherwise} \end{cases}$$

$$\mathrm{Var}(\log(1+R_{j,t})) = \begin{cases} \sigma^2 + \sigma_{\varepsilon,i}^2 & \text{if } j \text{ researched} \\ \sigma^2 + \sigma_{\varepsilon,i}^2 + \sigma_{\alpha,i}^2 & \text{otherwise} \end{cases}$$

- Structure $\rightarrow$ cannot learn about $1 + R_{M,t}$ from researching individual stocks
- Given no-shorting constraint, only hold stocks with $\hat{\alpha}_{i,j,t} > 0$
- CAPM intuition: all market log β's $= 1$, investors search for info about alphas

Notes on lognormal setting

- Could also use normal returns. In this case, shocks are additive.
- Normal returns are problematic.
- Good news: normal returns model gives same results

  Results/propositions independent of normal/lognormal distinction

The Investor's Problem

$$\max_{\{c_{i,t}\},\{s_{i,t}\},\{\omega_{\hat{\alpha}_t^i}\}} \mathrm{E}\left[\sum_{t=A_i}^{64} \beta^{t-A_i} \frac{c_{i,t}^{1-\gamma}}{1-\gamma}\right] + \mathsf{V}_{65}(W_{i,65})$$

$$s.t. \quad c_{i,t} + q_{i,t}s_{i,t} \leq W_{i,t}, \quad W_{i,t+1} = (W_{i,t} - c_{i,t} - q_{i,t}s_{i,t})(1 + R^p_{\hat{\alpha}_t^i}),$$

$$1 + R^p_{\hat{\alpha}_t^i} = \omega^{*\prime}_{\hat{\alpha}_t^i}(1 + \tilde{R}_{\hat{\alpha}_t^i}(\hat{z}_{i,t})), \quad \hat{z}_{i,t} \sim Poiss(s_{i,t}), \quad \omega^*_{\hat{\alpha}_t^i},\ q_{i,t}s_{i,t},\ c_{i,t} \geq 0.$$

- $\hat{z}_{i,t}$ is set of stocks encountered
- $c_{i,t}$ is period-t consumption
- $\hat{\alpha}_t^i$ is vector of learned alphas (length $\hat{z}_{i,t}$)
- $q_{i,t}s_{i,t}$ is research expenditure
- $R^p_{\hat{\alpha}_t^i}$ is (stochastic) portfolio return
- $\tilde{R}_{\hat{\alpha}_t^i}$ is vector of asset returns
- $\omega^*_{\hat{\alpha}_t^i}$ are optimal portfolio weights, conditional on $\tilde{R}_{\hat{\alpha}_t^i}$

Computational Burdens of Dynamic Model

- Dynamic model is computationally expensive
- For each level of wealth/costs/beliefs, need to find optimal level of research $(s_{i,t})$ and portfolio weights $(\omega^*_{\hat{\alpha}^i_t})$ for each t.
- Fortunately, two shortcuts exist:

1. Well known. With CRRA utility and stationary returns, portfolio choice is independent of time horizon
2. Turns out, static model well approximates dynamic research decisions

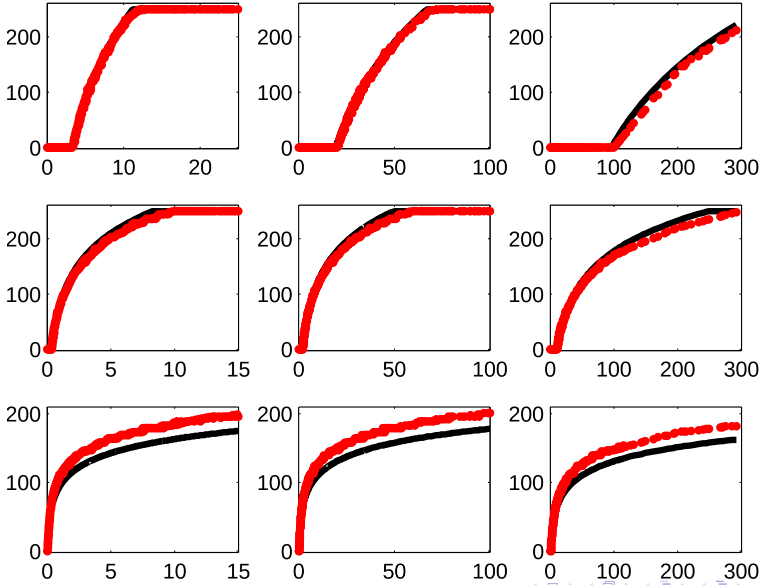
Consider the following Static Problem

$$\max_{s_i} \ \mathrm{E}\left[\frac{((W_{0,i} - q_i s_i) \times (1 + R^p_{\hat{\alpha}_i}))^{1-\gamma}}{1-\gamma}\right]$$

$$s.t. \quad 1 + R^p_{\hat{\alpha}_i} = \omega^{*\prime}_{\hat{\alpha}_i}(1 + \tilde{R}_{\hat{\alpha}_i}), \ \ \hat{z}_i \sim Poiss(s_i), \ \ \omega^*_{\hat{\alpha}_i} \geq 0, \ \ q_i s_i \leq W_{0,i}.$$

- $\hat{z}_i$ is set of stocks encountered
- $\hat{\alpha}_i$ is vector of learned alphas (length $\hat{z}_i$)
- $q_i s_i$ is research expenditure
- $R^p_{\hat{\alpha}_i}$ is (stochastic) portfolio return
- $\tilde{R}_{\hat{\alpha}_i}$ is vector of asset returns (including R and R_M)
- $\omega^*_{\hat{\alpha}_i}$ are optimal portfolio weights, conditional on $\tilde{R}_{\hat{\alpha}_i}$

Static vs Dynamic: Vert. axis = research, Horz. axis = wealth



Pursuing the Static Framework

- Because static model closely approximates dynamic model, will only solve and estimate static model
- Solution details for s_i and $\omega^*_{\hat{\alpha}_i}$ are covered in paper
- Keep in mind, static model is just a first-approximation for the dynamic framework

Parameterizing the Model

- Asset return data comes from CRSP Monthly Stock File
- Sample period: January 1970 - December 2010
- Use one-year ahead compounded returns for stocks in top 1,000 by market share in previous month.
- Annual-Nominal Returns.
- $463,618$ returns $\rightarrow \mathrm{E}[1+R_j]$ and $\mathrm{Var}(1+R_j)$
- Market fund is equal-weighted average of each return in given month-year
- 480 fund returns: $\rightarrow \sigma^2$ (recall $\mathrm{E}[1+R] = \mathrm{E}[1+R_j]$ by assumption)
- $\gamma = 4$

Parameter Values

Parameter	Description	Value
R	Risk-free rate	0.020
μ	$\mathrm{E}[\log(1+R_M)]$	0.107
σ^2	$\mathrm{Var}(\log(1+R_M))$	0.033
$V=\sigma^2+\sigma_\alpha^2+\sigma_\varepsilon^2$	$\mathrm{Var}(\log(1+R_j))$	0.165
γ	Risk Aversion	4
	Max $\#$ of Stocks Held	75

Summary of Model Results

1. The optimal level of research increases with wealth
2. The expected number of stocks held increases with research
3. Given research, the expected number of stocks held decreases with $\sigma^2_{\alpha,i}$
4. The expected fraction of total equity allocated to stocks increases with $\sigma^2_{\alpha,i}$
5. The expected fraction of wealth allocated to equity increases with $\sigma^2_{\alpha,i}$
6. The expected fraction of total equity allocated to stocks increases with number of stocks held
7. The expected fraction of total wealth allocated to equity (weakly) increases with number of stocks held

Result 1: The optimal level of research is increasing in wealth

- Model offers an optimal research condition:

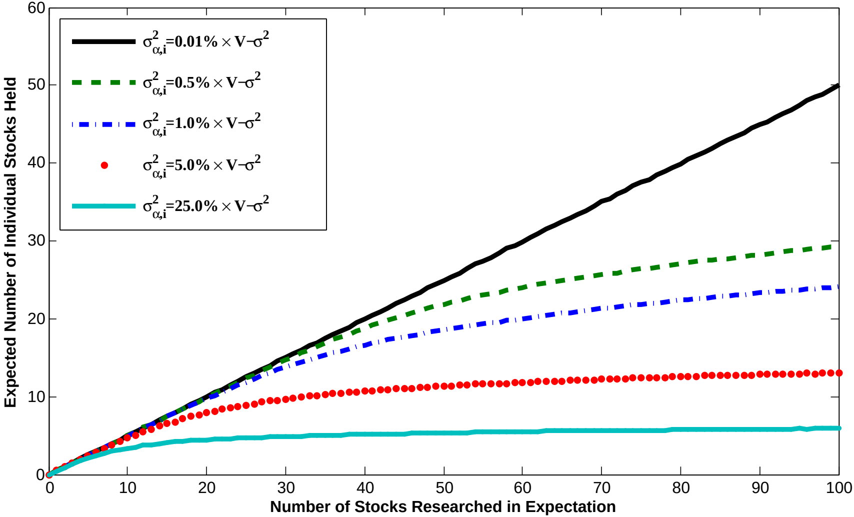
$$\frac{E[(1+R_{s+1})^{1-\gamma}]}{E[(1+R_s)^{1-\gamma}]} = \frac{(W_{0,i} - q_i s)^{1-\gamma}}{(W_{0,i} - q_i(s+1))^{1-\gamma}}$$

- The LHS approaches one as s increases, the RHS approaches one as $W_{0,i}$ increases (for $\gamma > 1$)

- This means $\widetilde{W}_{s,q_i,\sigma^2_{\alpha,i}}$ is increasing in s, $\rightarrow$ optimal level of *research* is increasing in $W_{0,i}$

LHS Approximation

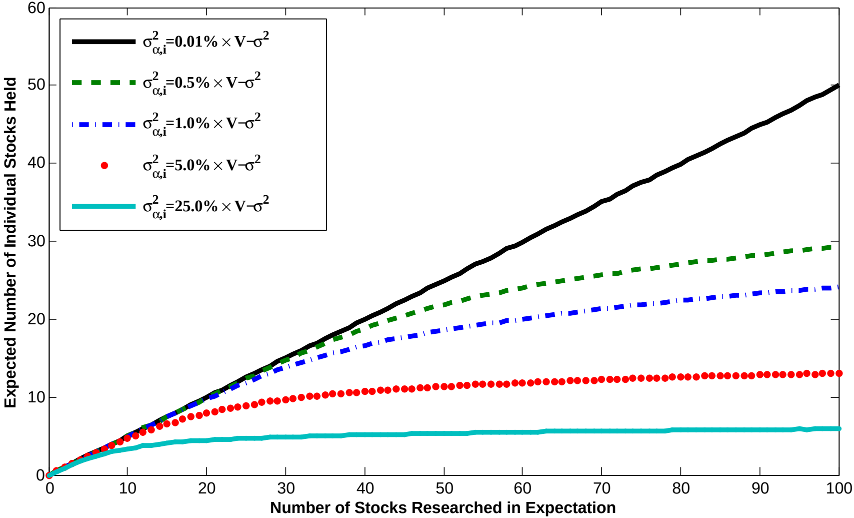
Result 2: The expected number of stocks held is increasing in research



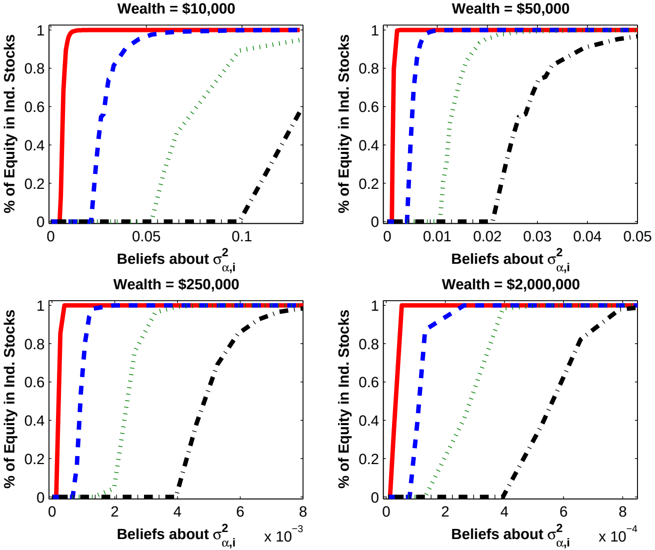
A note on research and beliefs

- Note: Research is NOT monotonically increasing in $\sigma^2_{\alpha,i}$. This is ok.
- Identification comes from joint distribution of $\#$ of stocks held and their allocation. Not from a one-to-one mapping of research and $\#$ of stocks held.

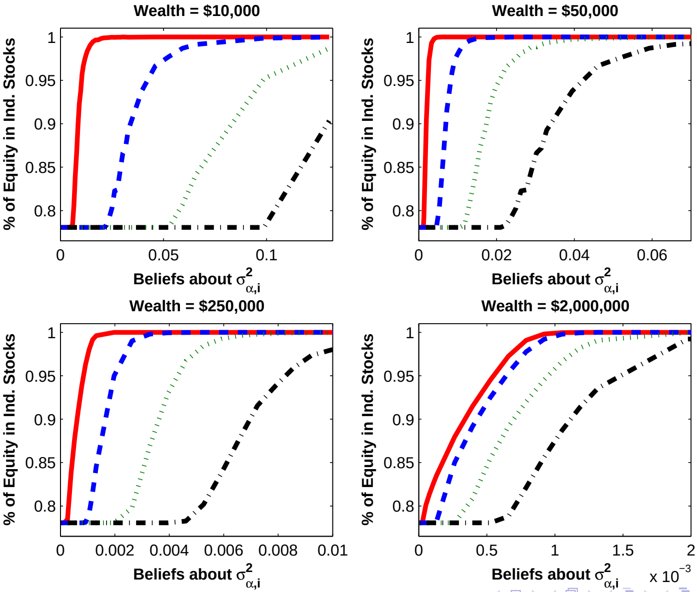
Result 3: Given research, the expected number of stocks held is decreasing in $\sigma^2_{\alpha,i}$



Result 4: The expected fraction of total equity allocated to stocks increases with $\sigma^2_{\alpha,i}$



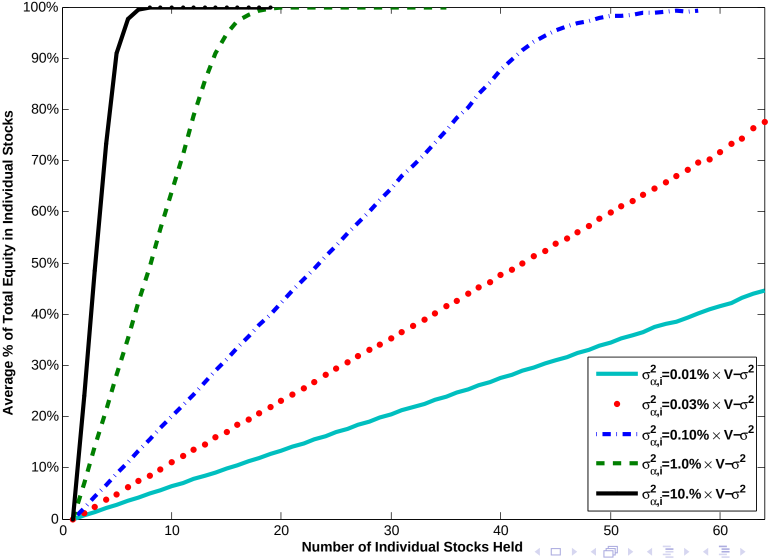
Expected fraction of wealth allocated to equity increases with $\sigma^2_{\alpha,i}$



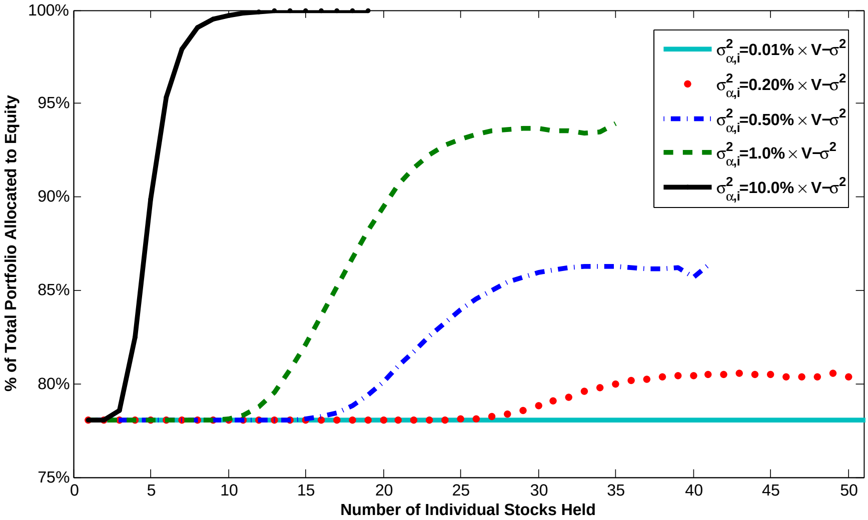
Confidence in Stock Picking

- Combined, Results 3-4 indicate more confident households:
  1. Hold fewer stocks (ceteris paribus)
  2. Invest higher fraction of equity in these stocks
- Results empirically supported: Ivkovic et. al. (2008) find more concentrated investors outperform more diversified investors

Result 5: Expected fraction of total equity allocated to stocks increases with the number of stocks held



Fraction of Total Portfolio Allocated to Equity (weakly) Increases with # of Stocks Held



Summarizing...

- Four stylized facts from the SCF:

1. Likelihood of holding individual stocks $\uparrow$ wealth
   - **Model $\rightarrow$ likelihood of holding ind. stocks $\uparrow$ wealth**
2. $\#$ stocks held $\uparrow$ wealth
   - **Model $\rightarrow$ $\#$ held $\uparrow$ wealth (independence needed)**
3. Fraction of total equity allocated to individual stocks $\uparrow$ $\#$ held
   - **Model is consistent with this fact (Result 5)**
   - Imposes restrictions on parameter estimates
4. Fraction of wealth allocated to equity $\uparrow$ $\#$ stocks held
   - **Model is consistent with this fact also**

Identification

- $4 + K$ parameters to estimate:
  - $\{\phi, \tau\} \rightarrow$ the proportion of non-market individual stock return variance that is predictable

    Recall: $\frac{\text{predictable variance}}{\text{non-market variance}} = \frac{\sigma^2_{\alpha,i}}{V - \sigma^2} \sim Beta(\phi, \tau)$
  - $\{\mu_q, \sigma_q^2, \beta\} \rightarrow$ the mean and variance of research costs q_i

    Recall: $q_i \sim logn(\mu_q + \beta Y_i, \sigma_q^2)$

Identification Cont.

- Beliefs: $\dfrac{\sigma^2_{\alpha,i}}{V - \sigma^2} \sim Beta(\phi, \tau)$
  - Identified from joint distribution of $\#$ stocks held and fraction of equity assets in stocks held
  - Low $\#$ held AND high proportion of equity assets invested $\Rightarrow$ $\dfrac{\sigma^2_{\alpha,i}}{V - \sigma^2} \sim Beta(\phi, \tau)$ is large
  - High $\#$ held OR low proportion of equity assets invested $\Rightarrow$ $\dfrac{\sigma^2_{\alpha,i}}{V - \sigma^2} \sim Beta(\phi, \tau)$ is small
  - In probabilistic sense: These statements are about likelihoods

Identification Cont.

- Research Costs: $\log(q_i) \sim N(\mu_q + \beta Y_i, \sigma_q^2)$
  - Identified from joint distribution of $\#$ stocks held, fraction of equity assets in stocks held, AND wealth
  - Low wealth, low allocation to stocks $\Rightarrow$ low research costs
  - High wealth, high allocation to stocks, low $\#$ held $\Rightarrow$ high costs
- Also identification value in non-stockholders

Maximize Sum of Individual Probabilities

- p_i is individual probability
- Based on:
  - Probability of investing ω_i in individual stocks given $\hat{z}_i$ held
  - Probability of holding $\hat{z}_i$ stocks given s_i^* (optimal research)
  - Search over $\{\phi, \tau, \mu_q.\sigma_q, \beta\}$ to maximize $\sum_i \log(p_i)$

Detailed Probability Function

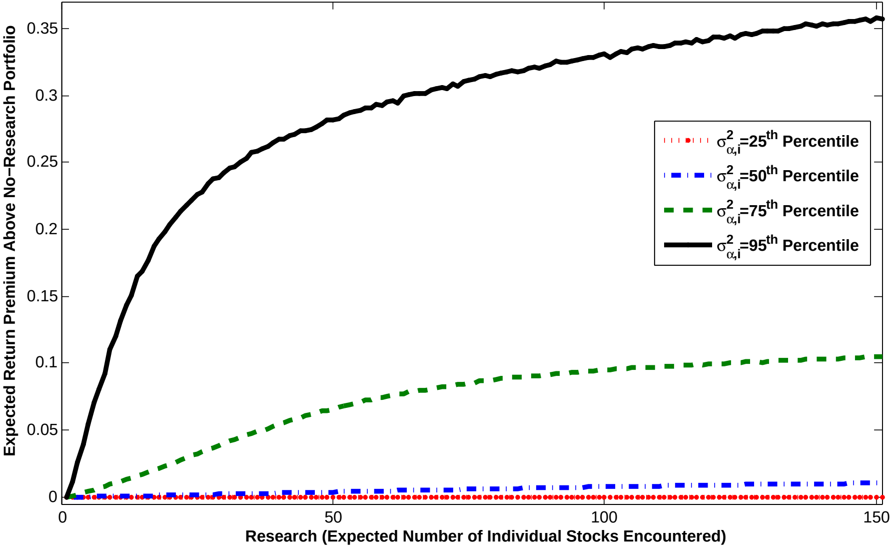
Results

Parameter	Estimate	Lower Bd.	Upper Bd.
ϕ	0.160	0.130	0.198
τ	7.337	5.393	10.008
μ_q	-5.367	-7.394	-3.215
σ_q	1.711	1.516	1.941
β_{inc}	-0.125	-0.242	0.000
β_{FA}	0.520	0.092	0.959
β_{ed}	-0.185	-0.308	-0.070
β_{age}	0.047	0.028	0.066

Results: Distribution of δ_i

- $\{\hat{\phi}, \hat{\tau}\} \rightarrow$ median value of $\dfrac{\sigma^2_{\alpha,i}}{V - \sigma^2} = .0012$
  - median household believes 12 basis points of total non-market variation is predictable
- 75th and 95th percentile values are 0.0167 and 0.1181, respectively
- Means 75% of population believes less than 2% of non-market variation is predictable

Expected Excess Return over the No-Research Portfolio



Comparison to Brokerage Data

- Beliefs may seem unreasonably optimistic
- Yet, Merkle (2013) finds investors' average *quarterly* outperformance is 2.89%
- 75^{th}, 90^{th} and 95^{th} percentiles of this outperformance are 5%, 15%, and 20% respectively.
- Expected excess returns estimated here are quantitatively similar to those elicited directly from brokerage respondents

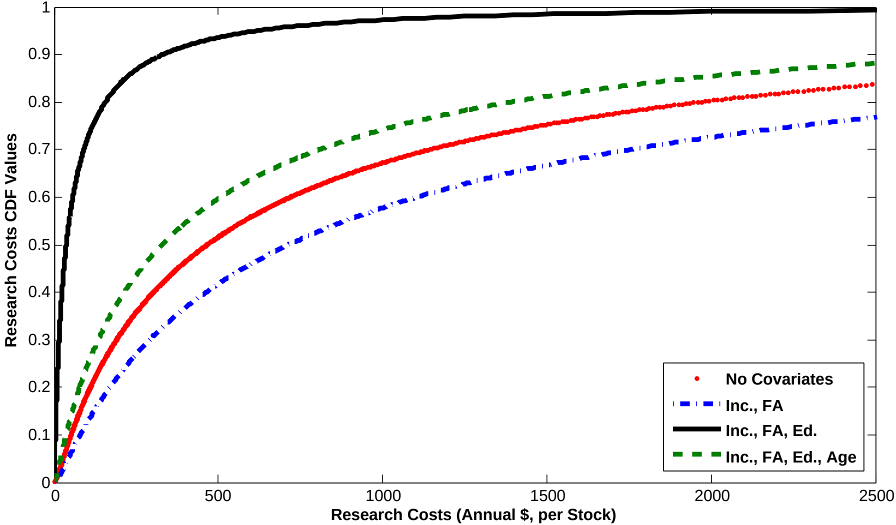
Jensen's Alpha in Actively Managed U.S. Equity Funds (from Glode 2011)

Decile (Alpha)	Alpha (%, per month)	Expenses (%)	Total Fee (%)
	Panel A. One-Factor Model		
1	-1.35	1.67	1.89
2	-0.51	1.52	1.77
3	-0.31	1.38	1.63
4	-0.19	1.35	1.60
5	-0.09	1.27	1.51
6	-0.01	1.23	1.44
7	0.09	1.23	1.46
8	0.22	1.34	1.58
9	0.42	1.35	1.50
10	1.21	1.45	1.65
1-10	-2.56	0.21	0.24
	[0.09]***	[0.04]***	[0.05]***

Results: Distribution of Research Costs

- $\{\hat{\mu_q}, \hat{\sigma_q}, \hat{\beta}\} \rightarrow$ Research costs q_i:
    - 25th percentile = $\$103.77$
    - median = $\$329.08$
    - 75th percentile = $\$1,043.60$
- Professonal financial advice and age raise costs
- Education lowers research costs

Research Costs CDF

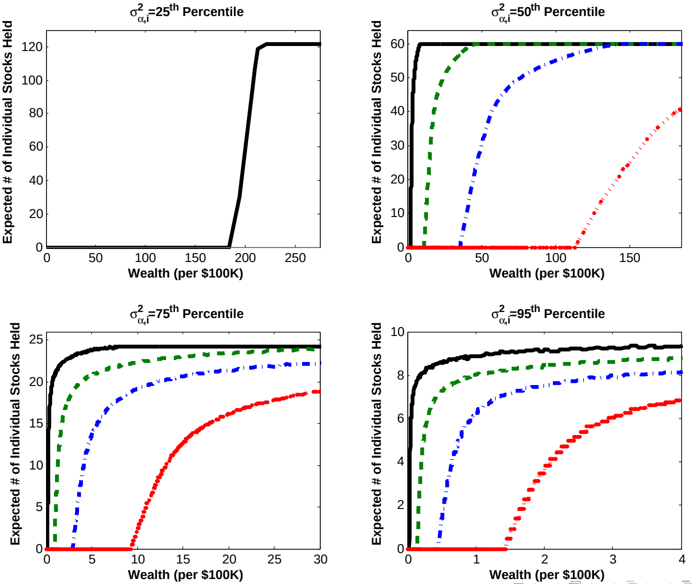


Research Costs

- Researching 10 stocks per year $\rightarrow$ nearly $\$3,500$ in annual research costs! (at median)
  - Most households don't hold any individual stocks (only 19.4% of weighted sample hold stocks)
  - 17% of those with 1-5 stocks invest over 90% of equity portfolio in those stocks.

    Research costs must be high to dissuade more research.
- q_i < 25th percentile more reasonable; 44% of low wealth stockholders (less than \$100k) with < 30% allocated to stocks

Expected number of Stocks Held



Concluding Remarks

- Structural model of costly research and household beliefs is identified only by wealth and portfolio choices
- Model can explain a number of stylized facts about household stock holdings
- 50th-75th percentiles of belief distribution expect to earn what top 2-3 active management deciles earn.
- Upper tail of belief distribution is ***REALLY*** optimistic; expect $> 35\%$ return premium from moderate research
- But beliefs can't be too crazy: many hold large number, and many hold none (even wealthy households)
- Research costs are large, but make sense given data

Data

- Survey of Consumer Finances: 1995, 1998, 2000, 2004 and 2007
- Inclusion criteria - start with 8,739 obs. after dropping missing data:
  - Drop households with $< \$1,000$ and $> \$30\text{MM}$ (2,480 obs.)
  - Drop ages < 22 or > 64 (1,400 obs.)
  - Drop households with stock in employer (includes family) (496 obs.)
  - Drop households with no equity, total equity > 100% (1,809 obs.)
  - Drop non-trading stock holders and "non-active" investors (787 obs.)

return

Fact 2 - # Ind. Stocks Held ↑ Wealth

Covar	**Dep Var = Number of Individual Stocks Held**						
Wealth	0.584***	0.513***	0.521***	0.513***	0.527***	0.527***	0.590***
	(0.054)	(0.080)	(0.076)	(0.073)	(0.074)	(0.074)	(0.108)
$(\text{Wealth})^2$	-0.002***	-0.002***	-0.002***	-0.002***	-0.002***	-0.002***	-0.002***
	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
Income	-	0.548	0.63	0.57	0.56	0.55	1.274
	-	(0.453)	(0.523)	(0.571)	(0.570)	(0.575)	(1.053)
Fin. Adv.	-	-	(0.309)	-0.646**	-0.500*	-0.522*	0.139
	-	-	(0.287)	(0.258)	(0.281)	(0.286)	(0.949)
Educ.	-	-	-	0.028	0.093***	0.089***	0.183**
	-	-	-	(0.028)	(0.029)	(0.028)	(0.087)
Age	-	-	-	-	-0.025***	-0.029***	0.010
	-	-	-	-	(0.009)	(0.009)	(0.040)
Own H.	-	-	-	-	-	0.309	-0.038
	-	-	-	-	-	(0.233)	(0.982)
Obs.	1,767	1,767	1,767	1,767	1,767	1,767	581
R^2	0.279	0.289	0.290	0.291	0.293	0.294	0.504

Return

Fact 2 Cont. - Distribution of Wealth by # of Stocks Held

# Stocks	Freq.	% of Obs	Mean Wealth	Min Wealth	Max Wealth
0	1,186	67.12	56,900	1,010	21,300,000
1	67	3.79	130,030	6,917	19,400,000
2	52	2.94	79,639	1,317	24,900,000
3	50	2.83	113,233	3,677	8,941,736
4	25	1.41	70,270	9,135	1,654,000
5	32	1.81	202,928	12,400	5,767,218
6	33	1.87	590,967	26,947	13,100,000
7	10	0.57	137,787	35,135	16,200,000
8	23	1.30	250,000	25,636	11,500,000
9	3	0.17	454,838	454,838	1,932,000
10	52	2.94	326,000	36,557	14,300,000
12	19	1.08	866,072	380,943	27,100,000
15	32	1.81	375,957	62,071	12,600,000
20	47	2.66	1,531,787	48,855	20,600,000
25	17	0.96	913,385	155,890	7,201,693
30	22	1.25	1,063,682	298,500	27,000,000
40	14	0.79	2,348,000	730,569	26,700,000
50	12	0.68	5,197,295	1,134,932	28,500,000
75	15	0.85	3,658,043	224,628	22,100,000

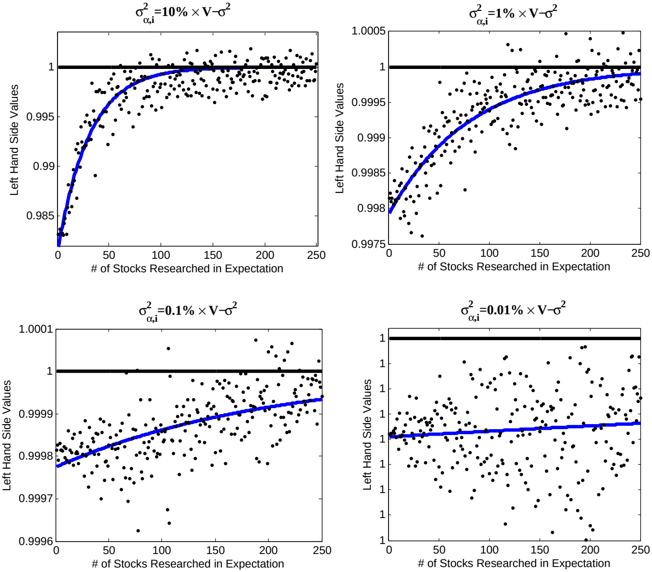
Return

LHS Approximation

- Take $\sigma^2_{\alpha,i} \in \alpha\text{-}grid$, s_i as given
- Draw value $\hat{z}_i \sim Poiss(s_i)$. Draw $\hat{\alpha}_i$ (vector).
- Calculate $\omega^*_{\hat{\alpha}_i}$. Gives port return $\log(1 + R_p)$
- From CDF, take values corresponding to $\{.00001, .0001, .0002, ..., .999\}$.
- Raise each to $(1 - \gamma)$, and average over CDF values using corresponding probabilities.
- Do this 7500 times and average. This gives $\mathrm{E}[(1 + R_s)^{1-\gamma}]$. Do this for all $s_i \in \{1, 2, ...s_{max}\}$.
- Gives $\dfrac{\mathrm{E}[(1 + R_{s+1})^{1-\gamma}]}{\mathrm{E}[(1 + R_s)^{1-\gamma}]}$ for all s_i.

Then Fit Negative Exponential Function

Return



Individual Probability Function

$$
\begin{aligned}
p_i &= \sum_{\widetilde{\sigma}^2_\alpha} \sum_{\widetilde{q}} \left[\sum_{z(s^*)} Pr(\omega^i_R | z(s^*), \hat{z}_i, \widetilde{\sigma}^2_\alpha) Pr(\hat{z}_i | z(s^*), \widetilde{\sigma}^2_\alpha) Pr(z(s^*)) \right] \\
&\quad \times Pr(\widetilde{q} | \mu_q, \sigma_q, Y_i, \beta) Pr(\widetilde{\sigma}^2_\alpha | \phi, \tau).
\end{aligned}
$$

- $Pr(\omega^i_R | z(s^*), \hat{z}_i, \widetilde{\sigma}^2_\alpha)$ is prob equity allocation to ind. stocks
- $Pr(\hat{z}_i | z(s^*), \widetilde{\sigma}^2_\alpha)$ is prob of holding $\hat{z}_i$ stocks given z encountered
- $Pr(z(s^*))$ is prob of encountering z given s_i
- $Pr(\widetilde{q} | \mu_q, \sigma_q, Y_i, \beta)$ is prob that i has cost q_i
- $Pr(\widetilde{\sigma}^2_\alpha | \phi, \tau)$ is prob that i has belief $\sigma^2_{\alpha,i}$ Return