Bank Overleverage and Macroeconomic Fragility[*]

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Abstract

This paper develops a dynamic general equilibrium model that explicitly includes a banking sector engaged in a maturity mismatch. We demonstrate that individual competitive banks take on excessive risks compared to the social optimum, resulting in overleverage and inefficiently high crisis probabilities. The model accounts for the banks' tendency to underestimate systemic risks in laissez-faire economies. The result calls for policy intervention to reduce the high crisis probabilities. To this end, the government can commit to bailing out banks through public supply of liquidity or a low-interest rate policy. As opposed to the intention of the government, however, expectations of a bailout could incentivize banks to be even more overleveraged, leaving the economy exposed to higher crisis probabilities.

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1 Introduction

Theories of banking have come a long way, elaborating on the social benefits spreading outward and the inherent fragility from within. In the meantime, macroeconomists have made a variety of attempts to incorporate realistic financial intermediaries – banks in particular – into dynamic general equilibrium models. A juxtaposition of the standard models of banking (Diamond and Dybvig 1983, Allen and Gale 1998, 2004) with a few dynamic macroeconomic models (Bernanke and Gertler 1989, Carlstrom and Fuerst 1997, Kiyotaki and Moore 1997) can clearly point to a yet-to-be filled gap between the micro-theory of banking and macroeconomics. The gap may be summarized as follows: many macroeconomic models have successfully incorporated a bilateral (i.e., non-bank/intermediary based) lender-borrower relationship and analyzed its consequences and implications for the macroeconomy while few macroeconomic models appear to have crystallized the roles and perils of banking systems in the light of a number of real-world experiences of financial crises. Banks in the real world, as articulated by micro-theories, provide unique services for their customers. They typically raise funds via short-term liabilities and invest them, in part, in illiquid assets, an action that is widely acknowledged as a maturity mismatch or maturity transformation.[1] In addition, as articulated by Allen and Gale (2007), micro-theories of banking have broadly emphasized other special elements in banking business, such as provision of liquidity insurance for depositors against liquidity (preference) shocks and inherent exposure to crisis risks. These special elements can be understood to be attempts to compensate for incomplete financial markets, but they are given short shrift in macroeconomics.[2]

This paper develops a dynamic general equilibrium model with a banking sector to address basic yet unsolved questions: does a banking sector with a maturity mismatch affect macroeconomic fluctuations? If yes, how could it improve or undermine economic welfare? We integrate a model of banking developed by Diamond and Rajan (2001a, 2012) into an overlapping-generations (OLG) model. The OLG model with a maturity mismatching banks is a general equilibrium model that

[1] The banks in our paper broadly refer to financial intermediaries that raise funds via short-term debt such as repo and commercial paper, and transform maturities on their balance sheet. Demand deposits are an extreme case of short-term debt.

[2] A number of recent studies have focused on the roles of banks in dynamic stochastic general equilibrium models. We will discuss them in Section 6.

explicitly includes factor markets. In particular, capital goods are saleable in the capital market. A key assumption is that individual banks do not internalize the effects of changes in capital prices on the solvency of the banking system as a whole. More precisely, while the banks internalize all the price changes of their own debt, they act as price-takers outside the deposit/liquidity market.

Our main finding is that a competitive banking sector tends to take on excessive systemic risks and gives rise to an inefficiently high probability of financial crises that result in a devastating contraction in macroeconomic activity. We introduce social planning (SP) banks that maximize the social welfare represented by households' expected utility and define the second-best optimum as the allocation that the SP banks achieve in an economy where full state-contingent securities are not available. Not surprisingly, the SP banks cannot achieve the first-best allocation because the lack of state-contingent securities leaves them with no choice but to face a strictly positive probability of a financial crisis. In addition to the unavoidable crisis risk faced by the SP banks, we find that the competitive banking sector in a laissez-faire (LF) economy underperforms compared to the SP banks in terms of social welfare, precipitating crises more frequently.

Generally, competitive banking systems, while they can compensate in part for market incompleteness, may not necessarily achieve an efficient risk exposure because of the pecuniary externalities in the LF economy. Diamond and Rajan (2001a, b) demonstrated that issuance of non-state-contingent short-term debt (e.g., demand deposits) by banks streamlines financial intermediation and promotes liquidity creation in economies where full state-contingent securities (i.e., Arrow securities) are not available. As mentioned, Diamond and Rajan's argument reconfirms that the benefit of banking business comes with a cost of potential insolvency of banks or financial crises, because banks rely on non-state-contingent debt rather than renegotiable financing. Banks need to strike the right balance between profitability and risks of insolvency, facing their own solvency constraints. As long as the price of an asset, which is held on their balance sheet, affects the banks' solvency constraint, the shadow value of the constraint for price-taking banks deviates from that of the SP banks. Pecuniary externalities arise because price-taking banks in the LF economy do not internalize the general equilibrium effects of a change in asset prices on the constraint against their decision regarding leverages.

Our model shares a similar kind of excessive risk-taking, which manifests in overleveraged banks in our model, with earlier research on pecuniary externalities that results in over-credit.[3] But the source of the pecuniary externalities in our paper differs from those of the earlier studies. Notably, the pecuniary externalities arise in our model as a result of incorporating the maturity-mismatching banks that are inherently subject to a solvency constraint, because they issue non-state-contingent debt. This is in sharp contrast to many early models that impose a borrowing constraint on particular investors. In sum, we argue that the two conditions – (i) issuance of non-state-contingent short-term debt by banks and (ii) their asset-price-dependent solvency – would result in underperformance of the LF banking system compared to the second-best allocation.[4]

The remaining question is why the LF banks give rise to overleverage and excessive risk-taking, rather than underleverage and under-credit. Suppose that the LF banks face a high liquidity shock that they all fall into insolvency. If each of them cuts their own leverage, their investment in illiquid assets would decline. Because the lower investment in illiquid assets reduces the capital good production, the price of capital increases. As a result, the solvency of the banking system as a whole would be consolidated by the increases in the capital prices. Although all the LF banks know the social benefit of collective deleveraging, such attempt is unlikely to be made individually, because of the lack of incentives in the LF economy.

When the banks determine their leverage, pecuniary externalities creates the wedge between the social and private marginal cost of increasing the leverage. If a single bank finds it optimal to raise its leverage, in the absence of heterogeneity by assumption, every other bank takes the same action based on the same idea. But the LF banks take other banks' behaviors as given to each other, and this systemic risk-taking results in higher risks of over-investment in the illiquid projects and, accordingly, over-supply of capital. The over-supply gives rise to lower asset prices, which erode the banks' solvency more quickly than in the social optimum.

Banks in the LF economy are not disregarding the danger of the synchronized behavior of

[3] A few examples include Bianchi (2010), Bianchi and Mendoza (2010), Jeanne and Korinek (2012a, b), Korinek (2010), Lorenzoni (2008), Mendoza (2010), and Stein (2012).

[4] Our model is in sharp contrast with that of Allen and Gale (1998) in which the maturity-mismatching LF banks always achieve the second-best allocation. Their model includes a single liquidity market where the banks can internalize all the effects through the single price changes. In contrast, our model includes multiple markets (i.e., the capital and labor markets) in some of which banks take prices as given.

the system as a whole. Simply, however, each LF bank is selfish. They lack in incentives to coordinate with each other and cannot collectively deleverage to the socially optimal level. Put another way, the pecuniary externality ill-incentivizes banks to take on excessive risks systemically. This coordination failure can pave the way to better understanding the weakness of LF banking system that have been observed repeatedly in past financial crises.

Furthermore, our work suggests several policy implications. A variety of policy measures to enhance the resilience of banking systems can be considered and, in fact, are currently being explored in the real world.[5] We assess the realistic policy measures from the viewpoints of crisis probabilities rather than pursuing an optimal policy design that achieves the second-best allocation. One highlight is that pre-announced bank bailouts may ill-incentivize banks to take on even higher risks, adding to the even higher crisis probabilities. A similar argument is likely to be applied to pre-commitment to a low interest rate policy that aims to prop up a banking system near crisis for the purpose of emergency liquidity provision. In principle, if banks are informed in advance that they would be bailed out by, for example, publicly supplied liquidity at a time of elevated market distress, they have little reason to be better prepared for an extremity through precautionary deleveraging. While public supply of liquidity *per se* has the effect of curbing crisis probability, banks' anticipation of such a bailout can reverse the expected outcome, resulting in more frequent crises. Finally, we argue that banks' capital requirement with prompt corrective action (PCA) may be less exposed to such risks of exacerbation of excessive risk-taking in the banking system.

The rest of the paper proceeds as follows. Section 2 illustrates the macroeconomic model with maturity-mismatching banks and characterize the banks' optimal leverage in the competitive equilibrium. In Section 3, we compare the LF equilibrium characterized in Section 2 with the allocation achieved by the SP banks and elucidate why a competitive banking sector tends to be overleveraged. Section 4 discusses numerically the probability of a financial crisis, the size of distortions, and the issue of crisis prevention along with comparisons with a number of empirical studies. Section 5 assesses policy measures aimed at reducing crisis probabilities and discusses a

[5] Globally coordinated efforts toward creating more stringent banking sector regulations can be observed in Basel III, which will be enacted stepwise in a few years. The U.K. government introduced a new bank levy, while the U.S. Congress approved the Dodd-Frank Act in 2010.

range of policy implications. Section 6 discusses relations with existing models. Section 7 concludes.

2 A Macroeoconomy with Banks

2.1 Agents, Endowment, Preferences, and Technology

We consider an infinite-horizon OLG model incorporating banks with a maturity mismatch. Each generation of agents consists of households, entrepreneurs, and bankers. Each period, generation t is born at the beginning of period t and lives for two periods, t and $t+1$. Each agent is identical and constant in the population. Furthermore, an initial old generation lives for one period and the subsequent generations live for two periods.

Households are risk averse and subject to a liquidity shock that affects their preference for consumption over the two periods. The liquidity shock is an aggregate shock and the only source of the uncertainty in the model. The households aim to smooth their consumption intertemporally. Following models in the theories of banking (e.g., Diamond and Rajan 2012), households are endowed with a unit of consumption goods at birth and do not consume the initially endowed consumption goods at the beginning of period t. The households deposit all initial endowments at banks operating in the same generation.[6] They receive wages w_t in the competitive labor market by supplying one unit of labor in both periods, t and $t+1$.

Entrepreneurs are risk neutral and have access to capital producing technology. They launch long-term investment projects at the beginning of period t, by borrowing households' endowments via the banks in the same generation. The investment project requires one period for gestation, and capital goods are produced in period $t+1$. We call this capital producing technology a "project." Entrepreneurs sell the capital goods in the competitive market for the capital goods price q_{t+1}.

Banks raise funds from households and lend them to entrepreneurs at the beginning of period t.[7] In principle, we follow Diamond and Rajan (2001a) to model banks. Banks are risk neutral and competitive at raising and lending funds in the markets. They issue demand deposits (short-term

[6] Following models of the theories of banking, we implicitly assume intra-period perishability of endowments. More precisely, all endowments perish before the realization of the liquidity shock.

[7] We assume intra-generational banking, which effectively means that all bankers of generation t die out at the end of period $t+1$.

debt) and commit to repaying the households. In the nature of demand deposits, banks can provide insurance against depositors' liquidity shocks. However, when households demand repayment before the completion of the entrepreneurs' projects, banks must liquidate premature projects to meet the demand for repayment. This maturity mismatch, represented by the combination of long-term assets and short-term liabilities, leaves banks exposed to risks of a default because, depending on the amount of withdrawals in the interim period, the banks' solvency is endangered.

The technology to produce consumption goods Y_t is represented by a standard constant-returns-to-scale Cobb-Douglas production function:

$$Y_t = F\left(K_t, H_t\right) = K_t^{\alpha} H_t^{1-\alpha},$$

where K_t and H_t denote the capital stock and hours worked, respectively. Demand for labor and capital satisfies

$$w_t = F_{H,t} = (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} \tag{1}$$

$$q_t = F_{K,t} = \alpha\left(\frac{K_t}{H_t}\right)^{\alpha-1}. \tag{2}$$

Accordingly, the second derivatives are denoted by $F_{KK,t}$, $F_{HH,t}$, and $F_{HK,t}$.

In what follows, we describe each agent's decisions (consumptions, withdrawal, and liquidation of the entrepreneurs' projects) *after* the liquidity shock is realized. Then, we move on to the bank's decision on their leverage *before* the realization of the liquidity shock. Table 1 summarizes the sequence of events in each generation.

2.2 Households

Under the competitive banking sector, each household accepts the banks' offer on deposit face value D_t at the beginning of period t, and observes the liquidity shock θ_t in the middle of period t. The liquidity shock is common across all households in the same generation and has the probability density function $f(\theta_t)$ with a support of $[0,1]$. This shock represents households' preference for consumption when young and signals the need for liquidity in period t.[8]

After the realization of θ_t, households make their decisions for consumption smoothing without uncertainty. Given that a crisis does not take place, households then choose withdrawal amount g_t to maximize

$$\begin{aligned} U\left(C_{1,t}, C_{2,t+1}\right) &= \theta_t \log C_{1,t} + (1-\theta_t) \log C_{2,t+1} \\ \text{s.t. } C_{1,t} &= w_t + g_t \\ C_{2,t+1} &= w_{t+1} + R_t \left(D_t - g_t\right), \end{aligned} \tag{3}$$

where $C_{1,t}$ and $C_{2,t+1}$ denote the consumption of households born in period t when young and old, respectively. Each household supplies a unit of labor in each period and receives wage income w_t in period t and w_{t+1} in period $t+1$. Here R_t denotes the one-period gross interest rate from period t to $t+1$.

In our model, a financial crisis takes place with the endogenous probability π_t, depending on the realization of θ_t. With the probability $1-\pi_t$, a financial crisis is not taking place and households can withdraw g_t in period t and all the remaining deposits in period $t+1$.[9] With the probability π_t, however, a financial crisis arises and households' withdrawals amount to the liquidation value of premature projects, $X\ (<1)$, in period t and nothing is left in period $t+1$. In the case of a crisis, households fail to smooth out their consumption and end up with $C_{1,t} = w_t + X$ and $C_{2,t+1} = w_{t+1}$.

When the households can smooth out their consumption, the intertemporal first-order condition for consumption is satisfied:

$$\frac{\theta_t}{1-\theta_t}\left(\frac{C_{1,t}}{C_{2,t+1}}\right)^{-1} = R_t. \tag{4}$$

[8] Although in fact all households are subject to the same aggregate shock, we assume that an infinitesimally small number of households are believed to face a different θ_t from other households. This assumption ensures the existence of a Nash equilibrium, in which all households run to the banks when households believe that the banks are insolvent under the observed θ_t.

[9] In the maximization problem of households, we assume that wage income in period t is low relative to the initial endowment, ensuring a non-negative withdrawal g_t in the equilibrium.

Given the Euler equation (4), the withdrawals in the absence of a crisis can be written as

$$g_t = \theta_t \left(\frac{w_{t+1}}{R_t} + D_t \right) - (1 - \theta_t) w_t. \tag{5}$$

The withdrawal function implies that large θ_t and D_t are likely to precipitate a financial crisis.

2.3 Entrepreneurs

Entrepreneurs are risk neural and maximize their expected lifetime utility represented by $E\left(C_{1,t}^e + C_{2,t+1}^e\right)$, where $C_{1,t}^e$ and $C_{2,t+1}^e$ denote entrepreneurs' consumption when young and old. They use a unit of consumption goods financed from banks for their capital goods production, and this production technology takes one period for gestation before its completion. In period $t+1$, the project yields a random capital goods output $\breve{\omega}$, which takes a value distributed over $[\omega_L, \omega_H]$ with the probability density function $h(\breve{\omega})$.[10] If this project is prematurely liquidated in period t, the transformation from the consumption goods into capital is incomplete. As a result, the output is reduced to X units of consumption goods and is repaid fully to banks in period t. When the project is completed in period $t+1$, however, entrepreneurs can sell their output in the capital goods market for the price represented by q_{t+1}. After repaying the banks, they are left with $1-\gamma$ of the share of their profit and enjoy their own consumption based on their linear utility. We assume that entrepreneurs are endowed with $\underline{I}$ units of capital goods at the beginning of period $t+1$.[11] They sell this endowment capital together with the newly created capital made from the consumption goods transferred from the households in period $t+1$.

2.4 Banks

Banks are also risk neutral and maximize their expected lifetime utility $E\left(C_{1,t}^b + C_{2,t+1}^b\right)$, where $C_{1,t}^b$ and $C_{2,t+1}^b$ denote consumption of banks when young and old. We borrow the microfoundation of the banking business from Diamond and Rajan (2001a, b): banks issue demand deposits (short-

[10] Following the literature, we take the assumption that there is no aggregate uncertainty in the project outcome.

[11] For simplicity, we assume a 100 percent depreciation rate in the law of motion for capital. The introduction of the endowment of capital goods here guarantees a finite capital price in the aftermath of a financial crisis in which all projects are scrapped due to full liquidation.

term debt) as a commitment device to compensate for the lack of transferability of their collection skills and to promote liquidity creation. As discussed in a number of preceding works on banking (such as Diamond and Dybvig 1983 and Allen and Gale 1998), the banks determine D_t before observing the liquidity shock, which is realized in the middle of period t.

Each bank has no initial endowment at birth but has a special skill to acquire knowledge about entrepreneurs' business. This skill enables a bank to act as the relationship lender that can develop an alternative, but less efficient, use of the entrepreneurs' project. As discussed in Diamond and Rajan (2001a, b, 2012), this knowledge enables the bank to acquire a fraction γ of the realized project's outcome in period $t+1$.[12]

We also follow earlier studies on the assumption for the distribution of entrepreneurs to which each bank makes loans. Each bank attracts many entrepreneurs through a competitive offer on the loan, resulting in an identical portfolio shared by all the symmetric banks. This setup effectively leads to a convenient outcome in the model: each bank and the aggregate economy face an identical distribution of entrepreneurs. In period t, the banks receive signals ω that perfectly predict the realized value of $\tilde{\omega}$ in period $t+1$. With this information ω and the households' liquidity demand observed in period t, each bank chooses one of the options: (i) to liquidate projects in period t, obtaining X of consumption goods per project; or (ii) to collect a fraction $\gamma q_{t+1}\omega$ from a completed project in period $t+1$. The bank liquidates the project if the outcome of a project falls short of $\tilde{\omega}_{t+1}$, defined as a function of R_t/q_{t+1}:[13]

$$\tilde{\omega}_{t+1} = \frac{X}{\gamma}\frac{R_t}{q_{t+1}}. \tag{6}$$

Otherwise, the bank continues the project, and then receives repayment of $\gamma q_{t+1}\omega$ and entrepreneurs consume the remaining fraction of outcome, $(1-\gamma)\, q_{t+1}\omega$, per project. After repaying the full amount of the households' withdrawals, the banks consume their own capital.

[12] We also assume that once a bank has loaned to entrepreneurs, no other lenders can replicate the collection skills.

[13] Equation (6) can be reinterpreted as follows: $\gamma q_{t+1}\omega/X$ corresponds to the marginal rate of transformation (MRT) between the period-t consumption goods (i.e., liquidation) and the period-$t+1$ consumption goods (i.e., continuation of projects). The MRT is here compared with the marginal rate of substitution of the households that is observed as the interest rate, R_t.

Let the banks' asset be $A\left(R_t/q_{t+1}\right)$. The banks' asset at the beginning of period t (i.e., prior to the withdrawals) can be expressed as

$$\begin{aligned} A\left(\frac{R_t}{q_{t+1}}\right) &= \int_{\omega_L}^{\tilde{\omega}_{t+1}} Xh\left(\omega\right)d\omega + \frac{\gamma q_{t+1}}{R_t}\int_{\tilde{\omega}_{t+1}}^{\omega_H} \omega h\left(\omega\right)d\omega \\ &= L\left(\frac{R_t}{q_{t+1}}\right) + \frac{\gamma q_{t+1}}{R_t} I\left(\frac{R_t}{q_{t+1}}\right). \end{aligned} \tag{7}$$

Note that $h\left(\omega\right)$ is interchangeable with $h\left(\breve{\omega}\right)$ owing to the perfect signaling. The bank asset denoted in (7) can be decomposed into two components: the values of the prematurely liquidated projects denoted as $L_t = L\left(R_t/q_{t+1}\right) \equiv \int_{\omega_L}^{\tilde{\omega}_{t+1}} Xh\left(\omega\right)d\omega$, which is used to meet the liquidity demand (i.e., withdrawals) from the households, and the banks' share of the investment output (measured in the present value of consumption goods) denoted as $\gamma q_{t+1} I_t/R_t$, where I_t is given by $I_t = I\left(R_t/q_{t+1}\right) = \int_{\tilde{\omega}_{t+1}}^{\omega_H} \omega h\left(\omega\right)d\omega$.

The banks are subject to the solvency constraint $D_t \leq A\left(R_t/q_{t+1}\right)$. Because banks' assets decrease when they are discounted by a high R_t and advance on high capital prices q_{t+1}, it can be shown that $A\left(\cdot\right)$ monotonically decreases with R_t/q_{t+1}. We can then define the relative price R_t^*/q_{t+1}^* that satisfies the solvency constraint with equality

$$D_t = A\left(R_t^*/q_{t+1}^*\right). \tag{8}$$

We refer to R_t^* and q_{t+1}^* as the *threshold* interest rate and capital price, respectively. Hereafter, we denote a variable with an asterisk as the variable on the threshold. For the purpose of subsequent discussion, we note that given $A\left(R_t/q_{t+1}\right)$, the bank leverage $D_t/(A_t - D_t)$ is uniquely determined once D_t is chosen, and hence we refer to D_t as leverage hereafter. We will discuss this issue in Section 4 in terms of numerical interpretation.

2.5 Market Clearing Conditions

Four markets need to clear in the competitive equilibrium: (i) liquidity; (ii) consumption goods; (iii) capital goods; and (iv) labor. The liquidity market clearing condition is given by

$$L\left(\frac{R_t}{q_{t+1}}\right) = \theta_t \left(\frac{w_{t+1}}{R_t} + D_t\right) - (1-\theta_t)\, w_t. \tag{9}$$

Next, the market clearing condition for consumption goods is

$$Y_t + L\left(\frac{R_t}{q_{t+1}}\right) = C_{1,t} + C_{2,t} + C^e_{2,t} + C^b_{2,t}. \tag{10}$$

The left-hand side of (10) includes the supply of goods from the liquidated projects. On the right-hand side of (10), $C_{2,t}$, $C^e_{2,t}$, and $C^b_{2,t}$ are consumption when generation $t-1$ is old.

The capital goods market clearing condition is

$$K_{t+1} = \begin{cases} \underline{I} + I\left(R_t/q_{t+1}\right) & \text{at normal times} \\ \underline{I} & \text{at crises.} \end{cases} \tag{11}$$

Here the equation suggests that the capital goods supply sharply declines, conditional on a crisis. Throughout the paper, we use $\underline{w}$ and $\underline{F}_H$ to denote the wage rate and the marginal product of labor evaluated at $K_{t+1} = \underline{I}$.

Finally, both young and old generations supply a unit of labor in each period. Therefore, H_t equals two for all t.

2.6 Optimal Bank Leverage

We now consider the banks' optimal choice for the leverage. The banks choose the size of their leverage before the realization of the liquidity shock. We focus on the laissez-faire (LF) banks in this subsection, and will discuss the social planning (SP) banks in Section 3.

The banks are competitive at issuing demand deposits, and we assume that households' endowments are scarce in comparison to entrepreneurs' projects. As a result of competition, the banks make a competitive offer of deposits for households, aiming to maximize the household welfare (Allen and Gale 1998, 2007), while in fact they are maximizing their own profits. Maximizing the household utility via the deposit offers means that banks internalize the liquidity market clearing condition in determining the offer. Through this internalization, the banks take into account possible changes in the crisis probability π_t. On the other hand, outside the liquidity market, they take the capital prices and wages as given.

To understand how the banks' choice of D_t affects π_t, we take three steps: First, we define a function R^*_{LF} as

$$R^*_{LF}(D_t) = q^*_{t+1} A^{-1}(D_t) \tag{12}$$

from (8). We emphasize that, in $R^*_{LF}(D_t)$, the threshold capital price q^*_{t+1} is treated as a parameter, reflecting the price-taking behavior of the LF banks. Second, using (9), we define a function θ^*_{LF} as

$$\theta^*_{LF}(D_t) = \frac{L\left(R^*_t / q^*_{t+1}\right) + w_t}{w_t + D_t + w^*_{t+1}/R^*_t}, \tag{13}$$

where $R^*_t = R^*_{LF}(D_t)$ while w_t, w^*_{t+1} and q^*_{t+1} are given parameters for the LF banks. We reemphasize that these prices are not constant parameters but in fact vary according to (1) and (2). But for the LF banks, when they determine the optimal D_t, they just take them as given. With $R^*_t = R^*_{LF}(D_t)$ and other threshold variables, the threshold level of the liquidity shock $\theta^*_t = \theta^*_{LF}(D_t)$ clears the liquidity market with the LF banks as shown in (13).

The final step is to connect θ^*_t to the crisis probability π_t. The above-defined $\theta^*_{LF}(D_t)$ means that any changes in D_t always give rise to changes in θ^*_t for the liquidity market to clear. By the solvency constraint with equality (8), any level of D_t, once chosen, determines the threshold relative price, R^*_t/q^*_{t+1}. Hence, θ^*_t can be interpreted as the liquidity shock on the brink of a financial crisis. Namely, when θ_t is strictly greater than θ^*_t, the banks turn out to be insolvent and a crisis is precipitated. Thus, the crisis probability π_t has a one-to-one relationship to θ^*_t via the probability density function $f(\theta_t)$:

$$\pi_t = \int_{\theta^*_t}^{1} f(\theta_t)\, d\theta_t. \tag{14}$$

In principle, the banks' choice of the leverage specifies R_t^*/q_{t+1}^* and this threshold relative price determines the threshold level of the liquidity shock θ_t^*, completing the link between the bank leverage and the crisis probability.

We are now ready to set up the optimization problem for the banks to determine the size of their leverage. In the problem, as discussed, banks take into account the endogenously changing θ_t^*.

Problem LF *In a laissez-faire economy, banks maximize the household expected utility*

$$\begin{aligned}&\max_{D_t} \int_0^{\theta_t^*} \left\{\theta_t \ln\left(w_t + L_t\right) + \left(1-\theta_t\right)\ln\left[w_{t+1} + R_t\left(D_t - L_t\right)\right]\right\} f\left(\theta_t\right) d\theta_t \\ &+ \int_{\theta_t^*}^1 \left[\theta_t \ln\left(w_t + X\right) + \left(1-\theta_t\right)\ln\left(\underline{w}\right)\right] f\left(\theta_t\right) d\theta_t,\end{aligned} \tag{15}$$

subject to (9) and (13).

The banks choose their leverage according to the following first-order condition:

$$\begin{aligned}&\left[\theta_t^* \log\left(\frac{\theta_t^* m_t^*}{w_t + X}\right) + \left(1-\theta_t^*\right)\log\left(\frac{R_t^*\left(1-\theta_t^*\right)m_t^*}{\underline{w}}\right)\right]\frac{d\pi_t}{d\theta_t^*}\theta_{LF}^{*\prime}\left(D_t\right) \\ =& \int_0^{\theta_t^*}\left[\frac{1}{m_t}\left(1 - \frac{w_{t+1}}{R_t^2}R_{LF}'\left(D_t,\theta_t\right)\right) + \left(1-\theta_t\right)\frac{R_{LF}'\left(D_t,\theta_t\right)}{R_t}\right] f\left(\theta_t\right) d\theta_t,\end{aligned} \tag{16}$$

where $m_t \equiv w_t + D_t + w_{t+1}/R_t$ is the lifetime income of households and $m_t^* \equiv w_t + D_t + w_{t+1}^*/R_t^*$, accordingly. More importantly, $\theta_{LF}^{*\prime}\left(D_t\right)$ is calculated from (13), taking capital prices and wages as given in line with the behavior of the LF banks. Likewise, with a slight abuse of notation, $R_{LF}'\left(D_t,\theta_t\right)$ denotes marginal changes in R_t with respect to D_t, given θ_t. By (9),

$$R_{LF}'\left(D_t,\theta_t\right) \equiv \frac{\theta_t}{L'/q_{t+1} + \theta_t w_{t+1}/R_t^2} > 0 \tag{17}$$

can be obtained where L' is the derivative of L_t with respect to R_t/q_{t+1}.

Equation (16) provides an economic interpretation that is in line with broad intuition. The terms in brackets on the left-hand side of (16) represent the loss of utility in a crisis compared to the threshold. From (14), the term outside the brackets indicates the marginal changes in a crisis probability with respect to bank leverage. The left-hand side of the equation consists of the expected loss of utility and the marginal change in the crisis probability. Simply put: the left-hand side of (16) is the marginal cost of increasing D_t.

The right-hand side of (16) consists of the effects of increasing leverage on the expected households' utility through their lifetime income. On the one hand, the increase in D_t has an outright positive effect on the households' income: the higher the leverage, the larger the withdrawal, allowing households to enjoy more consumption. On the other hand, the increase in D_t leads to a higher interest rate via liquidity shortage, discounting the households' labor income in period $t+1$ and reducing returns on forgoing withdrawal until period $t+1$. Hence, as far as the outright effect on the lifetime income exceeds the effect on the interest rate, the higher leverage is beneficial to households. Simply put: the right-hand side of (16) is the marginal benefit of increasing D_t.

Hereafter, we maintain the assumption that the outright effect of D_t, which raises household income, exceeds the discounting effect by a higher interest rate. Otherwise, discussing banks in the economy is totally pointless. With this assumption in place, strict concavity of (15) in D_t is the sufficient condition for the uniqueness of Problem LF. Intuitively, (15) could be regarded as a smooth concave function against D_t because, by and large, (15) is a linear combination of log functions while the strict concavity of (15) requires a few technical assumptions.[14] In fact, we later show that, in numerical computations, the equilibrium is unique where the marginal cost schedule is increasing in D_t and, conversely, the marginal benefit is decreasing in D_t at around the equilibrium.

We finally define the equilibrium in the LF economy as follows.

Definition (Laissez-faire economy) *A competitive equilibrium consists of allocations and prices* $\{g_t, D_t, L_t, K_t, I_t, H_t, R_t, q_t, w_t\}_{t=0}^{\infty}$ *such that (i) withdrawal decisions are given by (5) for* $\theta_t \leq \theta_t^*$*; (ii) banks' leverage satisfies (16); (iii) banks' liquidity supply is determined by (6); and (iv) all markets clear.*

[14] Technically, it is straightforward to confirm that neither $D = X$ nor $D = \infty$ could be the optimal levels of D_t. First, suppose $D = X$. Then the marginal cost of increasing D_t is strictly zero because a crisis is strictly a zero probability event. This means that raising D_t slightly above X is always a better choice. On the other extreme, suppose that $D = \infty$. Then, a crisis takes place for any θ_t, which implies that $D = \infty$ cannot be the optimum. As long as (15) is strictly concave and the two extreme values for D_t are not optimal, it is assured that the optimal bank leverage D_t lies in $[X, \infty)$. The details of the sufficient conditions for the uniqueness are available upon request.

3 Systemic Risks and Welfare

3.1 Social Planning Banks

In the previous section, banks maximize households' expected utility as a result of the perfect competition. In this section, however, we *assume* that banks can choose allocations as the constrained social planner. We call such banks the SP banks. Under this assumption, we demonstrate that the competitive equilibrium, defined in the previous section, cannot replicate the social optimum that will be characterized in this section. To lead off the analysis, we clarify the constraint to which the SP banks are subject. They must make all their decisions before observing θ_t. After realizing θ_t, they are left with no options. In other words, the SP banks are subject to the constraint that they can neither control households' behaviors nor choose their outright consumption levels because households can react to any realized value of θ_t. This assumption is made for an explicit reason. While we examine the social planner's problem in this section, nonetheless, we aim primarily to see the constrained optimum rather than the unconstrained optimum. The unconstrained, first-best optimum is conceptually easy to understand. By assuming that banks (or anyone else) can issue the Arrow securities that pay off contingent on all possible realizations of θ_t, households can enjoy the maximum utility without experiencing any financial crisis. But as we already discussed, banks are, by definition, entities engaged in a maturity mismatch and issue non-state-contingent pre-committed debt (e.g., demand deposits). Otherwise, banks are no longer banks and should be regarded as other types of financial intermediaries (e.g., private equity funds). Because in our model we cannot disregard the special element in banking business – a maturity mismatch – we assume that banks pre-commit to payment on their debt regardless of the states realized following their commitment. The extra ability given to the SP banks compared to the price-taking competitive banks is that the former can internalize all price effects in all markets when they make decisions regarding their leverage.

The SP banks do not take the factor prices as given, but take into account their changes reflecting the marginal product. Formally, we replace q_t and w_t with $F_{K,t}$ and $F_{H,t}$, respectively. Note that, nonetheless, the SP banks take the households' behaviors as given, as they cannot make their contract contingent on θ_t. In other words, households always choose their consumption and withdrawal given the pre-committed D_t by the banks.

To specify the problem for the SP banks, we clarify the solvency constraint with which the SP banks are faced, $D_t \leq A\left(R_t/F_{K,t+1}\right)$. We note that the constraint effectively remains the same as in Problem LF because $q_{t+1} = F_{K,t+1}$ from (2). The newly introduced solvency constraint for the SP banks, however, has different effects on the threshold because (8) and (12) are now replaced with

$$D_t = A\left(\frac{R_t^*}{F_{K,t+1}^*}\right) \tag{18}$$

$$R_{SP}^*\left(D_t\right) = F_{K,t+1}^* A^{-1}\left(D_t\right), \tag{19}$$

respectively in the problem for the SP banks where the SP banks can internalize general equilibrium effects of the factor prices.

We summarize the SP banks' problem as follows:

Problem SP *The social planning banks maximize the household expected utility,*

$$\max_{D_t} \int_0^{\theta_t^*} \left\{\theta_t \ln\left(F_{H,t} + L_t\right) + \left(1-\theta_t\right) \ln\left[F_{H,t+1} + R_t\left(D_t - L_t\right)\right]\right\} f\left(\theta_t\right) d\theta_t$$
$$+ \int_{\theta_t^*}^1 \left[\theta_t \ln\left(F_{H,t} + X\right) + \left(1-\theta_t\right) \ln \underline{F}_H\right] f\left(\theta_t\right) d\theta_t,$$

subject to

$$L\left(\frac{R_t}{F_{K,t+1}}\right) = \theta_t\left(\frac{F_{H,t+1}}{R_t} + D_t\right) - \left(1-\theta_t\right) F_{H,t} \tag{20}$$

$$\theta_{SP}^*\left(D_t\right) \equiv \frac{L\left(R_t^*/F_{K,t+1}^*\right) + F_{H,t}}{F_{H,t} + D_t + F_{H,t+1}^*/R_t^*}, \tag{21}$$

where $R_t^* = R_{SP}^*\left(D_t\right)$ *from (19) and* $\theta_t^* = \theta_{SP}^*\left(D_t\right)$ *from (21).*

Note that all the factor prices, including q_{t+1}^* and w_{t+1}^* in Problem LF, are replaced with marginal products in Problem SP. More importantly, because the SP banks factor in all general equilibrium effects, $\theta_{SP}^{*\prime}(D_t)$ can be denoted as $d\theta_t^*/dD_t$. The solution of Problem SP is conceptually comparable to the constrained optimum as discussed in Allen and Gale (1998).

In Problem SP, all the factor prices in period $t+1$ are functions of K_{t+1} or K_{t+1}^*. In this context, the capital goods market clearing condition in Problem SP needs extra attention when we replace the capital price with the marginal product in (11). We note that K_{t+1} depends solely on the market interest rate R_t and this relationship is denoted by a function $\Phi(R_t)$. Provided that a crisis does not take place, K_{t+1} evolves according to

$$\begin{aligned} K_{t+1} &= \underline{I} + I\left(\frac{R_t}{F_{K,t+1}}\right) \\ &\equiv \Phi(R_t), \end{aligned} \tag{22}$$

where $\Phi' < 0$ represents the derivative of K_{t+1} with respect to R_t.[15] To move on, we reaffirm that $F_{K,t+1}^*$ and $F_{H,t+1}^*$ in (18), (19), and (21) can be written as

$$\begin{aligned} F_{K,t+1}^* &= F_K\left(K_{t+1}^*, H\right) = F_K\left[\Phi\left(R_t^*\right)\right] \\ F_{H,t+1}^* &= F_H\left(K_{t+1}^*, H\right) = F_H\left[\Phi\left(R_t^*\right)\right]. \end{aligned}$$

Our first main result is as follows. With the factor prices replaced by marginal products in Problem SP, the allocations that the SP banks achieve differ from those achieved by the LF banks because of the extra ability given to the SP banks. Comparison between Problems LF and SP confirms that the two problems are subject to exactly the same constraints. With identical constraints, any discrepancy in the first-order conditions generally results in different allocations across the two problems. To see this, for example, we focus on $\theta_{SP}^{*\prime} = d\theta_t^*/dD_t$ and $\theta_{LF}^{*\prime}$, both of which are the part of the first-order conditions in each problem:

$$\theta_{SP}^{*\prime}(D_t) = \frac{1}{m_t^*}\left[\frac{\partial}{\partial R_t^*}\left(L_t^* - g_t^*\right) - \zeta_\varepsilon^*\right]\frac{dR_t^*}{dD_t} - \frac{\theta_t^*}{m_t^*}, \tag{23}$$

$$\theta_{LF}^{*\prime}(D_t) \equiv \frac{1}{m_t^*}\left[\frac{\partial}{\partial R_t^*}\left(L_t^* - g_t^*\right)\right] R_{LF}^{*\prime}(D_t) - \frac{\theta_t^*}{m_t^*}, \tag{24}$$

[15] Solving $\Phi' = \left(1 - \Phi' R_t F_{KK,t+1}/F_{K,t+1}\right) I'/F_{K,t+1}$ for Φ' ensures that Φ' is negative.

where $\zeta_{\varepsilon}^{*} \equiv (\Phi^{*\prime}/R_t^*) \left[\left(R_t^*/F_{K,t+1}^* \right)^2 L^{*\prime} F_{KK,t+1}^* + \theta_t^* F_{HK,t+1}^* \right]$ and $dR_t^*/dD_t = R_{SP}^{*\prime}(D_t)$. Alongside, $\Phi^{*\prime}$ and $L^{*\prime}$ in ζ_{ε}^{*} represent $\Phi'(R_t^*)$ and $L'\left(R_t^*/q_{t+1}^*\right)$, respectively.

In general, non-zero ζ_{ε}^{*} ensures the difference between the two equilibria, while, in fact, comparison of the two first-order condtions reveals extra discrepancies in addition to ζ_{ε}^{*}. The next subsection focuses on the key discrepancy between $R_{LF}^{*\prime}(D_t)$ and $dR_t^*/dD_t = R_{SP}^{*\prime}(D_t)$, which provides clear economic interpretation.

3.2 Crisis Probabilities and Marginal Systemic Risk (MSR)

We introduce a useful measure that facilitates assessment of the systemic risk of an economy, that is, marginal systemic risk (MSR), as the marginal increase in the crisis probability against a target variable at and around the equilibrium. MSR can be applied in broad models where a financial crisis takes place as a non-zero probability event. Depending on the focus of studies, MSR can be defined vis-à-vis bank leverage, aggregate credit, bank lending, or potentially, asset prices. In our model, MSR is defined with respect to a unit change in bank leverage. Specifically, let $D_{LF,t}$ be the level of bank leverage chosen in the LF economy. Then,

$$\begin{aligned} MSR_{k,t} &= \frac{d\pi_t}{d\theta_t^*}\theta_k^{*\prime}(D_{LF,t}) \\ &= -f(\theta_t^*)\,\theta_k^{*\prime}(D_{LF,t}) \quad \text{for } k = LF, SP. \end{aligned} \tag{25}$$

Recall that a bank in the LF economy takes other banks' decisions as given, but in fact the crisis probability is affected by the synchronized decisions by the banking sector as a whole. In this regard, $\theta_{LF}^{*\prime}(D_{LF,t})\,d\pi_t/d\theta_t^*$ can be interpreted as the marginal risks perceived by the individual price-taking banks, which can be contrasted with the true marginal risk, $\theta_{SP}^{*\prime}(D_{LF,t})\,d\pi_t/d\theta_t^*$ at the chosen $D_t = D_{LF,t}$. Technically, MSR is applicable for any level of D_t. We utilize MSRs evaluated at $D_t = D_{LF,t}$ because this allows us to directly compare the MSRs across different problems under the same allocations and prices.

With this interpretation in mind, we stress that, if $MSR_{LF,t}$ is smaller than $MSR_{SP,t}$, the gap indicates that the banking sector in the LF economy underestimates the marginal cost of higher leverage by not taking into account the systemic risks. As a result, the LF banks are likely to be overleveraged at (and around) the LF equilibrium. Instead, if $MSR_{LF,t}$ takes a larger value than $MSR_{SP,t}$, the gap points to underleverage in the LF economy. In principle, an undervaluation of $MSR_{LF,t}$ compared to $MSR_{SP,t}$ would provide the ground for government intervention to rein in excessive leverage of banks, because such regulatory risk reduction can improve welfare. Relying on the concept of the MSR, the second main result is that $MSR_{SP,t}$ exceeds $MSR_{LF,t}$. For the formal proof, our second main result requires one technical condition:

Condition 1 $\alpha\theta_t^* w_{t+1}^*/R_t^{*2} \leq (1-\alpha) L^{*\prime}/q_{t+1}^*$ *for* $D_t = D_{LF,t}$

Condition 1 ensures that the supply curve is steeper than the demand curve with respect to R_t^* in the liquidity market. In the condition, the right-hand-side indicates the slope of the demand curve for liquidity while the left-hand-side points to that of the supply curve. Condition 1 also ensures $\zeta_\varepsilon^* \geq 0$ for a chosen D_t. Now, we are ready to state Proposition 1.

Proposition 1 *Under Condition 1,* $MSR_{SP,t}$ *is strictly larger than* $MSR_{LF,t}$*.*

Proof. See Appendix A.1. ■

Proposition 1 provides a foundation for understanding why the crisis probability is higher in the LF economy than in the social optimum. From (23) and (24),

$$\begin{aligned} MSR_{SP,t} - MSR_{LF,t} &= f\left(\theta_t^*\right)\left[\theta_{LF}^{*\prime}\left(D_{LF,t}\right) - \theta_{SP}^{*\prime}\left(D_{LF,t}\right)\right] \\ &= \frac{f\left(\theta_t^*\right)}{m_t^*}\left\{\frac{\partial}{\partial R_t^*}\left(L_t^* - g_t^*\right)\left[R_{LF}^{*\prime}\left(D_{LF,t}\right) - R_{SP}^{*\prime}\left(D_{LF,t}\right)\right] + \zeta_\varepsilon^*\right\}. \end{aligned} \tag{26}$$

Note that all functions are evaluated at the LF equilibrium (i.e., $D_t = D_{LF,t}$). In (26), $f\left(\theta_t^*\right)/m_t^*$ and the slope of the excess liquidity supply function denoted as $\partial\left(L_t^* - g_t^*\right)/\partial R_t^*$ are both positive. Condition 1 ensures that we are left with the deviation of the changes in R_t^* with respect to D_t. From (12) and (19), the inverse function theorem yields

$$\begin{aligned} R_{LF}^{*\prime}\left(D_{LF,t}\right) - R_{SP}^{*\prime}\left(D_{LF,t}\right) &= \left(1 - \frac{1}{1 - F_{KK,t+1}^*\Phi^{*\prime}R_t^*/q_{t+1}^*}\right)\frac{q_{t+1}^*}{A^{*\prime}} \\ &= -\frac{R_t^*}{q_{t+1}^*}F_{KK,t+1}^*\Phi^{*\prime}\frac{dR_t^*}{dD_t} > 0, \end{aligned} \tag{27}$$

where $A^{*\prime} = A'\left(R_t^*/q_{t+1}^*\right)$. The sign of $dR_t^*/dD_t = R_{SP}^{*\prime}\left(D_{LF,t}\right)$ is assured to be negative as shown in Appendix A.1.

Clearly, (27) indicates that an individual price-taking bank underestimates the marginal changes in R_t^* due to the pecuniary externalities. An increase in leverage reduces R_t^*, because, in general, highly leveraged banks would be more likely to default under a lower interest rate. But the perceived reduction in R_t^* is different between the LF and SP banks and this gap creates the wedge in the two MSRs in our model. In this regard, this wedge reflects the LF banks' underestimation of the marginal cost of higher leverage. As a result of the underestimation of the marginal cost, the LF banking system find themselves insolvent more frequently than expected.

To better understand the gap in the MSRs, we can focus on $F_{KK,t+1}^*\Phi^{*\prime}$ included in (27). This term points to a side effect arising from higher leverage: in general equilibrium, the reduction in R_t^* increases $K_{t+1}^* = \Phi\left(R_t^*\right)$. That is, the lower R_t^* stimulates capital supply on the threshold and this increase in K_{t+1}^* triggers the decline in the threshold capital price q_{t+1}^*via the lower marginal product of capital. With this side effect, the lower capital price further undermines the bank's solvency, compared to the case without the side effect of increasing the leverage. Because the atomistic banks do not take into account this side effect, the lower-than-expected capital price and the undermined banks' solvency raise the probability of a financial crisis compared with the economy with the SP banks.

Looking at the real-world experience of past financial crises, it may be pointed out that, with hindsight, outlooks regarding asset prices frequently tended to be overly optimistic in the run-ups to crises. Some argue that such over-optimism arises from irrationality. While we do not claim that irrational behavior is irrelevant, our model suggests that despite the full rationality, pecuniary externalities can result in seemingly irrational over-optimism. The key to understanding the externalities lies in the synchronized decisions made by the individual banks in a competitive sector. For each bank, capital prices are given but the given prices affect the solvency of the banking system as a whole. In general, maturity-mismatching banks face solvency constraints because they issue non-state-contingent debt. As long as the effects of the asset prices on their solvency are not internalized, distortion arises. In our model, the distortion shows up as the overleveraged banking sector with a higher crisis risk because of the side effect as shown in (27), which can be interpreted broadly in line with the real-world observations.

4 Numerical Results

4.1 Solving the Model

Analytical results in the previous sections can be translated into numerical examples. We provide numerical solutions of the model in this section to address the following quantitative questions: (i) How frequently does a financial crisis arise? (ii) To what extent do the LF banks deviate from the social optimum? (iii) What can the calibrated model tell us about crises that should be prevented by government intervention in the banking sector? And (iv) How can we compare the numerical results with existing empirical studies on the probability of crises?

Our calibration mostly follows Diamond and Rajan (2012). We set the value of prematurely liquidated project X at 0.95. The distribution of entrepreneurs' projects is assumed to be uniform over $[\omega_L, \omega_H] = [0.5, 3.5]$, similar to the original calibration of Diamond and Rajan (2012). The degree of banks' special collection skills γ is set at 0.9. In addition to parameterization of Diamond and Rajan (2012), we need to set several additional parameters. We calibrate the capital share in the production function, α, to be 1/3. The capital goods endowment received by entrepreneurs, $\underline{I}$, and the level of total factor productivity, which is suppressed in (1) and (2), together effectively determine the size of the scarring effect of a financial crisis. We parameterize them so that the post-crisis contraction in output matches the estimated size of scarring effects from past empirical studies.[16] More importantly, we assume that the liquidity shock θ_t follows the beta distribution with a mean of 0.50 and a standard deviation of 0.07. This parameterization indicates a symmetric bell-shaped distribution. To simulate the model, we numerically solve the nonlinear system of the equations consisting of the first-order conditions and resource constraints.

Before interpreting the numerical results, we reconfirm the economic interpretations of D_t. In the context of our model, D_t represents the pre-committed gross return on bank deposits. On

[16] See Barro (2009) as discussed in Section 4.5.

the other hand, D_t cannot be translated into an annual percentage rate or an interest rate *per annum*, because the model does not specify the length of each period of time (e.g., one year or one quarter). To focus more clearly on the economic interpretations, D_t needs to be translated into a timeless measure such as the bank leverage. It is the exact reasoning that we have relied on in this interpretation of D_t, that is, bank leverage.

4.2 The Probability of Financial Crises

A notable feature of the model is that the probability of a financial crisis varies endogenously. A primary quantitative question to be addressed here is how frequently a financial crisis can take place.

Our simulation results are summarized in Table 2. The upper panel of the table reports that the LF banks take on more risks than the SP banks, indicating a higher crisis probability. We note that the two economies share the same state variable (i.e., the initial capital stock K_t) in our comparisons. Our calibration points to a 6.59 percent crisis probability in the LF economy compared to 4.50 percent in the social optimum. Hence, the results suggest that in an arbitrary period out of 1,000 simultaneous attempts, about 66 attempts would trigger crises in the LF economy. The overleverage can be confirmed by the values of D_t in the upper panel of the table. In fact, D_t is 1.2 percentage points higher in the LF economy than in the social optimum. Figure 1 plots the level of the expected utility against D_t, reflecting all the endogenous changes in factor prices (i.e., the social welfare of the households). Our numerical example indicates that the LF banks choose their leverage uniquely, shown at point B in the figure. In line with our main results, however, the LF banks cannot achieve the constrained optimum. In fact, our computation results reaffirm that this is the case.

4.3 The Size of Distortions

We next examine the extent to which the LF banks deviate from the allocation achieved by the SP banks. More broadly, we discuss the quantitative implications of the higher leverage in the LF economy for welfare.

We have shown in Proposition 1 that the LF banks in our model generate a strictly lower MSR than the SP banks under Condition 1. If we remove the condition, it could be argued that the proposition may not always be the case. Our simulation results confirm that the MSR with the LF banks, in fact, takes a smaller value than with the SP banks under a plausible parameter set. The upper panel of Table 2 reports the MSRs evaluated under the allocation in the LF economy. The deviation of the two MSRs is 0.45, indicating a higher risk in the LF economy and the extra increase in the crisis probability arises solely from pecuniary externalities. In particular, each LF bank makes the individually rational assessment that, if leverage is increased by 1 percentage point, the LF banks would be exposed to an extra increase in crisis probability of 1.64 percent, but the true increase in the probability is 2.12 percent.[17] Figure 2 also confirms $MSR_{LF,t} < MSR_{SP,t}$. The red and blue lines in the figure show how probabilities are perceived by the LF and SP banks around $D_{LF,t}$, respectively. The figure confirms that the red line is flatter than the blue line, reflecting the deviations in the MSRs, as predicted by Proposition 1.

Figure 3 plots the marginal cost and benefit in the two economies. The marginal cost is represented by solid lines, and the marginal benefit by dashed lines. The intersections A and B represent the choices of leverage under the social optimum and the LF economy, respectively. This numerical computation reconfirms that banks' underestimation of the marginal cost leads to overleverage. In this computation, we also observe a relatively small shift in the marginal benefit curve across the two economies, which slightly exacerbates the banks' overleverage.[18]

The lower panel of Table 2 compares the bank capital ratio defined as $(A_t - D_t)/A_t$ and the output of the consumption goods Y_{t+1} under the LF economy and the social optimum, when the realized value of θ_t takes the mean of 0.5. The LF banks are undercapitalized by 1.1 percentage point compared to the social optimum. Nevertheless, it may be surprising that the production does not substantially differ across the two allocations, provided that a financial crisis does not take place. We also compute the levels of consumption for households in a generation. The household's

[17] These probabilities are obtained by transforming the MSRs into the semi-elasticity of the probability.

[18] This shift results mainly from the LF bank's overestimation of wages in period $t+1$, conditional on a financial crisis not taking place. A marginal increase in leverage leads to greater consumption by households and a higher interest rate. A higher interest rate implies a lower amount of the completed project (i.e., lower capital accumulation). The lower capital accumulation also reduces the marginal product of labor, resulting in lower wages in period $t+1$. Since the LF banks disregard this income-reducing effect, they overestimate the marginal benefit of increasing D_t.

consumption is $(C_{1,t}, C_{2,t+1}) = (2.21, 2.61)$ in the LF economy in comparison to $(C_{1,t}, C_{2,t+1}) = (2.21, 2.59)$ in the social optimum.

The above exercise indicates that the welfare loss primarily arises from the inefficiently elevated crisis probability. Given that crises are considered to be rare events that we cannot observe frequently, the inefficiency or welfare loss may not be easily detected by looking at the volatility of consumption or output in normal times. In this sense, assessing the inefficiency with crisis probabilities or MSRs appears to be more appropriate than using the volatility of consumption or output.

4.4 Crisis Prevention

The prediction that the LF banking sector is overleveraged implies that the LF economy undergoes crises created by pecuniary externalities, some of which could be avoided under the social optimum. To illustrate this, we run the model over 100 periods by generating liquidity shocks randomly. Figure 4 plots the dynamic paths of output Y_t and θ_t^* under the two equilibrium allocations. The red line corresponds to the case of the LF economy, while the blue line points to the case of the social optimum.

The upper panel of Figure 4 shows that the dynamic paths of the output are almost identical except that the production under the LF economy sharply declines more frequently. Crises take place in periods 5, 16, and 94, and output falls sharply in each subsequent period. We note that this simulation result indicates that the latter two crises could have been prevented if the banks had taken the risks at the optimal level, implying a need for government intervention to forestall the crises. However, the first crisis takes place even in an economy in which the SP banks strike the right balance between the costs and benefits of increasing the leverage. Therefore, this crisis should not be avoided as discussed in Allen and Gale (1998) in the context of the optimal financial crises.

To better understand how differences arise in the two economies, the lower panel of the figure shows two different dynamic paths of θ_t^*. Recall that θ_t^* is defined as the threshold value of the liquidity shock that satisfies the solvency constraint on the brink of a default. This threshold level of θ_t^* is always lower in the LF economy, implying that the solvency constraint is tighter and the economy is more exposed to the risk of financial crises. The dashed line represents the realized θ_t in the simulations, which is identical across the two economies. The realized θ_t exceeds both high and low θ_t^* in the first crisis and reaches only the lower θ_t^* in the last two crises. Although the difference in the realized θ_t appears to be fairly small among the three crises, relative to the volatility of θ_t, the small difference in the liquidity shock affects the economic performance, through the risk-taking of the banking sectors.

4.5 Comparison with Empirical Studies

The numbers included in Table 2 may be compared to some recent empirical studies on crises. Among a number of works on catastrophe risks, Barro (2009) provides a comparable benchmark. He sets the disaster probability at two percent per year, arguing that a disaster could reduce GDP by 30 percent on average. Another notable example is Reinhart and Rogoff (2008, 2009). They document the frequencies of banking crises of over sixty countries using an expansive data going back to 1800s. Based on their dataset, they calculate the frequencies are 7.2 percent for the advanced economies. In a similar context, the Basel Committee on Banking Supervision (2010, BCBS) provides broader perspective on the frequency of banking crisis based on the multiple datasets, such as Laeven and Valencia (2008) and Reinhart and Rogoff (2008), and summarizes that the frequency lies in the range of 3.6 to 6.8 percent.

BCBS (2010) reports straightforward empirical measures that are comparable to the MSR as it assesses impacts of changes in bank capital on the probability of systemic banking crises. They estimate that a one percent increase in bank capital from the pre-reform cross-country average level could reduce the crisis probability by 1.0-1.6 percentage points.[19] We can underscore the proximity of the BCBS estimates and the numerical results included in Table 2.[20] Beyond such simple numer-

[19] Its estimates are based on multiple empirical methodologies, but it could be said that, primarily, reduced-form econometric models, such as probit/logit models, are used to produce those estimates. However, its estimates roughly match our simulated value (1.64) of the extra increase in the crisis probability from raising the leverage computed from the MSR for the LF economy.

[20] In addition to the evident proximity of the marginal changes in the crisis probabilities, the level of bank capital in the BCBS does not substantially differ from that in our model. The BCBS argues that the pre-reform cross-country average of the TCE/RWA (tangible common equity divided by Basel II risk-weighted assets) ratio is 7 percent. The

ical comparisons, we emphasize that MSR can be applied in line with a broad empirical exercise as typically demonstrated by the BCBS. Using reduced-form econometric models, a marginal increase in the crisis probability can be evaluated at around a certain point in the data. But empirical estimates, in general, cannot provide information on how distant the economy is located from the social optimum. Assessing to what extent the observed equilibrium could be improved by policy interventions may be of more interest. To this end, a comparison of empirically estimated marginal changes in crisis probabilities with MSRs calculated using structural models would promote discussion on the desirable size and design of regulations in the banking sector down the road.

5 Policy Intervention

This section discusses a variety of policy measures that have been implemented and are about to be enacted by both national and international regulatory bodies. In this regard, rather than examining the maximization of the social welfare, whose function is not specified, we focus on assessing the policies that aim to reduce the crisis probability by curbing banks' leverage D_t.

5.1 Bank Levy

Suppose that, in attempt to decrease the crisis probability, a government/central bank (GC) introduces a levy on bank size measured by its liability (D_t).[21] In practice, the levy can be used to rescue troubled banks by means of a bailout and, in fact, later we will consider such interventions with the intentions of a bank bailout. For the moment, in this subsection, we do not specify the purpose for which the GC spends the funds earned from the levy, but we simply assume that the GC consumes the levy (i.e., they just waste it) to focus on the impact of such a levy *per se* on banks' risk-taking behavior. The GC imposes τ percent of the levy on bank's liabilities. Under the levy in place, τD_t of tax burden, which is measured by period-t consumption goods, falls on the

TCE is an extremely narrow definition of bank capital that, by and large, could be doubled or even tripled (i.e., to 14 - 21 percent) if measured in more conventional measures for bank capital, such as the Tier I ratio.

[21] For example, the U.K. government enacted a bank levy as of January 2011 in an attempt to "encourage banks to move away from risky funding models" and to share the burden of financial crises with the banking sector. See HM Revenue and Customs (2010). Another example is the Volcker plan, which includes a proposal to restrict the size of banks' liabilities.

banks of generation t. In terms of operation, we assume that the GC collects $R_t \tau D_t$ of consumption goods at $t+1$ from banks. In period $t+1$, the GC consumes the collected consumption goods for itself, which is denoted by $C^g_{t+1} = R_t \tau D_t$. Because there is no uncertainty in the economy after the realization of θ_t, both banks and households correctly recognize that the banks' solvency is undermined by τD_t as of period t. Under the bank levy, the solvency constraint is written as

$$(1+\tau) D_t \leq A \left(R_t / q_{t+1} \right). \tag{28}$$

Accordingly, we replace (8) with

$$(1+\tau) D_t = A \left(R_t^* / q_{t+1}^* \right), \tag{29}$$

where all variables with an asterisk are redefined in line with the new constraint (28). From (29), we can define the threshold interest rate function R^*_{BL},

$$R^*_{BL} \left(D_t \right) = q^*_{t+1} A^{-1} \left[(1+\tau) D_t \right]. \tag{30}$$

In a similar manner to Problem LF, note that by the inverse function theorem, $R^{*\prime}_{BL} \left(D_t \right) = q^*_{t+1} (1+\tau) / A^{*\prime}$. Using (9), a function θ^*_{BL} can be defined as

$$\theta^*_{BL} \left(D_t \right) = \frac{L \left(R_t^* / q^*_{t+1} \right) + w_t}{w_t + D_t + w^*_{t+1} / R^*_t}, \tag{31}$$

where $R^*_t = R^*_{BL} \left(D_t \right)$ from (30) while w_t, w^*_{t+1} and q^*_{t+1} are given parameters for the competitive banks.

We now formally state the banks' problem with the levy as follows:

Problem BL *Let $\theta^*_t = \theta^*_{BL} \left(D_t \right)$. In an economy with a bank levy (BL), banks maximize (15) subject to (9) and (31).*

Table 3 reports the allocation and the crisis probabilities under a reasonable range of $\tau = 0.01, 0.02$, and 0.03. In comparison to the leverage of 1.061 in the LF economy (Table 2), banks' overleverage is reined in to 1.052, 1.044, and 1.036 for each case as confirmed in the first column ("bank levy") in Table 3. But the crisis probabilities rise, rather than decline, contrary to the expected outcome. The results can be interpreted easily. The effects of the levy acts via two channels: (i) the banks de-leverage in response to a levy and this channel in fact reduces crisis probabilities because the bank's active, intentional risk-taking subsides; and (ii) on the other hand, the levy erodes the banks' profitability and capital, which are tabulated in the fourth row of each panel. By law, the banks pay out money to their depositors and creditors out of the after tax profit and assets. This simply exposes the banks to a higher risk of insolvency, because they are left with fewer resources that can be paid out to their creditors. On balance, the latter effect dominates the former one, resulting in the higher crisis probabilities despite the banks' lower leverage. The point is that the GC does not rescue banks near crisis by using the collected tax, and this assumption regarding how the levy is spent is admittedly less realistic in the light of past experience and current practice.

5.2 Public Liquidity Provision as a Bank Bailout

5.2.1 Characterization

In the previous case, we assumed that the GC consumes collected tax for its own benefit. But, presumably and more realistically, the bank levy would be used for the particular purpose of bailing out troubled banks. As assumed in the previous case (BL), likewise, the GC collects $R_t \tau D_t$ of the levy in period $t+1$. Furthermore, in this case, the GC pre-commits to stepping into the liquidity market to provide the liquidity M_t to rescue a banking system near crisis in period t when such intervention is possible and needed. This emergency liquidity provision can prop up the banking system near crisis if properly designed with feasible financing.

In this case, we assume that the emergency liquidity provision by the GC is funded by tax collected from households after the realization of θ_t in period t. We further assume that the tax burden on households at t will be compensated by the income transfer from the GC at $t+1$ together with the interest payment.[22] Because, as noted, the GC collects $R_t \tau D_t$ of consumption goods from

[22] This assumption rules out inter-generational income transfers from the GC.

banks as the bank levy in period $t+1$, the GC can make the income transfer that keeps the lifetime income of households unchanged as long as the total supply of public liquidity does not exceed the amount of the bank levy, i.e., $\tau D_t \geq M_t$. Put differently, as long as the funding resource of the GC is assured, the income transfer can increase the total supply of liquidity by the amount of M_t in the period-t liquidity market.[23]

5.2.2 Implementation: commitment to a low interest rate policy

Because this intervention helps increase liquidity supply in the market, the interest rate would decline to a lower level than would have been the case otherwise. Consequently, the emergency liquidity provision at a time of elevated financial tension can be reinterpreted as a commitment to a low interest rate policy by a central bank.

The sequence is as follows. First, the GC announces its commitment to forestalling banks' insolvency by stepping into the period-t liquidity market to provide public liquidity in a case where the banking system cannot remain solvent without such an operation. Then, by fully recognizing the GC's commitment, the banks determine their leverage (D_t). Subsequently, the GC (and everyone else) recognizes the maximum interest rate (R_t^*) in line with (29) above which the banking system fails to remain solvent. Then, θ_t is realized. Suppose that the materialized θ_t exceeds a certain level θ_t^c. The high θ_t accordingly would give rise to a high interest rate, $R_t > R_t^*$, if the intervention did not take place. In line with the pre-announced commitment, however, the GC provides liquidity to prop up a banking system near crisis by cutting the market interest rate.[24] Such liquidity provision can effectively be reinterpreted as placing a cap on the market interest rate at $R_t \leq R_t^*$. At normal times, the liquidity market clearing condition in period t is given by (9) for $\theta_t \leq \theta_t^c$, where θ_t^c is formally defined as the level of liquidity preference shock that requires the GC to intervene in the

[23] An alternative way to validate the same operation is to simply assume that the GC has a storage technology for consumption goods between before and after the realization of θ_t. If this is the case, the GC can collect τD_t of consumption goods from banks of generation t before θ_t is realized and uses the resource to finance the intervention when possible and needed after θ_t is realized. If a high θ_t is realized, the GC provides $M_t\,(\leq \tau D_t)$ of liquidity out of the τD_t of funds that was set aside.

[24] From the viewpoint of the implementation, the GC makes a commitment to a level of R_t^* while the commitment is effectively equivalent to a level of the relative price R_t^*/q_{t+1}^*. This is because q_{t+1}^* has a one-to-one relationship with R_t^*, through $q_{t+1}^* = F_K\left[\Phi\left(R_t^*\right)\right]$.

market to supply extra liquidity and keep the interest rate at R_t^*:

$$\theta_t^c = \frac{L\left(R_t^*/q_{t+1}^*\right) + w_t}{w_t + D_t + w_{t+1}^*/R_t^*}. \tag{32}$$

By contrast, similar to the BL economy, we define $\theta_t^* = \theta_{BB}^*(D_t)$ that precipitates a financial crisis.

$$\theta_{BB}^*(D_t) = \frac{L\left(R_t^*/q_{t+1}^*\right) + w_t + \tau D_t}{w_t + D_t + w_{t+1}^*/R_t^*}, \tag{33}$$

where $R_t^* = R_{BB}^*(D_t)$. Here, $R_{BB}^*(D_t)$ is identical to $R_{BL}^*(D_t)$ because the banks' solvency constraint remains the same as (28). Note that if $\theta_t > \theta_t^*$, a financial crisis cannot be forestalled by the GC's intervention. For $\theta_t \in (\theta_t^c, \theta_t^*]$, the GC steps in and rescues a banking system near crisis by emergency liquidity provision M_t. Accordingly, the liquidity market clearing condition when the GC's intervention is underway is given by

$$\theta_t\left(\frac{w_{t+1}^*}{R_t^*} + D_t\right) - (1-\theta_t)\,w_t = L\left(R_t^*/q_{t+1}^*\right) + M_t. \tag{34}$$

In other words, the GC's liquidity provision is performed subject to the response function:

$$M_t = g_t^* - L\left(R_t^*/q_{t+1}^*\right), \tag{35}$$

with the maximum level $\tau D_t \geq M_t$, because the GC can finance this intervention only with the funds raised via the bank levy. Notably, this maximum level τD_t is independent of θ_t, because it is determined before observing θ_t.

We formally state the banks' problem under this policy intervention:

Problem BB *Let* $\theta_t^* = \theta_{BB}^* (D_t)$. *In an economy with the bank bailout (BB), banks maximize*

$$\max_{D_t} \int_0^{\theta_t^c} \{\theta_t \ln (w_t + L_t) + (1-\theta_t) \ln [w_{t+1} + R_t (D_t - L_t)]\} f(\theta_t) d\theta_t$$
$$+ \int_{\theta_t^c}^{\theta_t^*} \left\{\theta_t \ln (w_t + L_t^*) + (1-\theta_t) \ln \left[w_{t+1}^* + R_t^* (D_t - L_t^*)\right]\right\} f(\theta_t) d\theta_t$$
$$+ \int_{\theta_t^*}^{1} [\theta_t \ln (w_t + X) + (1-\theta_t) \ln (\underline{w})] f(\theta_t) d\theta_t, \quad (36)$$

subject to (9), (34), (32), and (33).

In this problem, the competitive banking sector takes capital prices, wages, and the GC's response function (35) as given. The objective function includes three terms. The newly included term reflects the household's expected utility with a banking system near crisis for $\theta_t \in (\theta_t^c, \theta_t^*]$ while the GC successfully bails out the system with the public liquidity provision. Reflecting the new term in the objective function, the efficiency condition for the banks with respect to D_t accordingly has the new term for the range of θ_t.[25]

Before moving on to the main results of this section, we emphasize that if $\tau = 0$, the allocations and prices both in the BL and BB economies are identical to the LF equilibrium. Bearing this fact in mind, the following proposition summarizes the main results regarding crisis probabilities across the BL and BB economies.

Proposition 2 *Let* $MSR_{BL,t}$ *and* $MSR_{BB,t}$ *be the marginal systemic risks in Problems BL and BB, respectively. Then, around the LF allocation,* $MSR_{BL,t}$ *is strictly larger than* $MSR_{BB,t}$.

Proof. An infinitesimally small $\tau > 0$ affects each MSR as follows:

$$\begin{aligned} MSR_{BL,t} &= -f(\theta_t^*) \theta_{BL}^{*\prime} (D_{LF,t}) \\ &= -f(\theta_t^*) \left\{ \frac{1}{m_t^*} \left[\frac{\partial}{\partial R_t^*} (L_t^* - g_t^*) \right] R_{BL}^{*\prime} (D_{LF,t}) - \frac{\theta_t^*}{m_t^*} \right\} \\ MSR_{BB,t} &= -f(\theta_t^*) \theta_{BB}^{*\prime} (D_{LF,t}) \\ &= -f(\theta_t^*) \left\{ \frac{1}{m_t^*} \left[\frac{\partial}{\partial R_t^*} (L_t^* - g_t^*) \right] R_{BB}^{*\prime} (D_{LF,t}) - \frac{\theta_t^* - \tau}{m_t^*} \right\}. \end{aligned}$$

[25] See Appendix A.2 for the details.

Recall $R_{BL}^{*\prime}(D_{LF,t}) = R_{BB}^{*\prime}(D_{LF,t}) = (1+\tau)\, q_{t+1}^{*}/A^{*\prime}$. With $D_t = D_{LF,t}$, the allocations L_t^*, g_t^*, m_t^*, and θ_t^* are the same across the two economies. Hence,

$$MSR_{BL,t} - MSR_{BB,t} = \frac{\tau f(\theta_t^*)}{m_t^*} > 0,$$

which proves the proposition. ■

As discussed in the comparison between the SP and the LF economies, likewise, a smaller MSR in the BB economy suggests that the crisis probability would be higher than in the BL economy. This can be interpreted in line with economic intuition: the GC intends to forestall a crisis by reining in the otherwise rising interest rate to a low level R_t^*. If the intervention succeeds, the banking system remains solvent even if it faces a high $\theta_t \in (\theta_t^c, \theta_t^*]$ with the aid of the GC. But this is not the end of the story. When the banks determine their D_t, they fully anticipate that the bailout will be enacted at a time of financial distress. By correctly taking into account the increased safety owing to the bailout (i.e., emergency liquidity provision), the banks take on more risks, resulting in a higher leverage, given the same burden levied on the banks. The ultimate outcome would be an even higher probability of crisis compared to an economy without such a bailout. Public liquidity provision as a bank bailout can thus raise, rather than reduce, the probability of a crisis as articulated in Table 3.

In a similar context, repercussions of authorities' commitment to a low interest rate policy via liquidity provision are pointed out by Diamond and Rajan (2012) and are discussed from a broader perspective in Rajan (2010).[26]

5.3 Capital Requirement with Prompt Corrective Action

In an attempt to reduce the crisis probability, the GC may choose another option: in fact, the capital requirement with prompt corrective action (PCA) has been up and running as the primary banking sector regulatory tool, as typically represented by Basel II. The regulation requires banks

[26] Rajan (2010) argues the idea using the widely acknowledged term in financial markets, the "Greenspan put." Specifically, "'Don't bother storing cash or marketable assets for a rainy day. We [the Fed] will be there to help you.' . . . it [the Fed] implicitly encouraged bankers to borrow short-term while making long-term loans, confident the Fed would be there if funding dried up. Leverage built up throughout the system."

to hold a certain minimum level of capital. If a bank fails to maintain the required level of capital, it would be taken into receivership by the GC. Because of the capital requirement, the banks are faced with the constraint, $\phi \leq \left[A\left(R_t/q_{t+1}\right) - D_t\right]/A\left(R_t/q_{t+1}\right)$, where ϕ points to the required minimum capital ratio. The requirement can equivalently be rewritten as the *PCA activation condition,*

$$D_t \leq (1-\phi)\, A\left(R_t/q_{t+1}\right), \tag{37}$$

which appears, on surface, similar to (28) although it is effectively quite different. Basically, (37) is not a solvency constraint for banks, because, even if the constraint is violated, banks may still hold positive capital and remain solvent. If a bank fails to hold $\phi A\left(R_t/q_{t+1}\right)$ of capital, it is taken into receivership by the GC. Under the PCA, the bank can continue to operate but it is run by new management, typically appointed by the GC.

An issue that emerges in such cases is that the new management appointed by the GC may have inferior skills in fostering the remaining long-term projects because they are less experienced bankers left with unfamiliar projects. Reflecting their inferior skills, we assume that, once the bank is taken into receivership, the new management of the bank can obtain $\lambda\gamma\omega \leq \gamma\omega$ from the completed project. If λ is equal to one, the bank's ability as a relationship lender is fully retained while, by contrast, if λ is strictly lower than one, it points to a loss of human capital in the banks because of the receivership.

The banks' assets under the PCA can be expressed as

$$\tilde{A}\left(R_t/q_{t+1};\lambda\right) = \int_{\omega_L}^{\omega_{t+1}^a} X h(\omega)\, d\omega + \frac{\lambda\gamma q_{t+1}}{R_t}\int_{\omega_{t+1}^a}^{\omega_H} \omega h(\omega)\, d\omega,$$

where we denote $\tilde{A}\left(R_t/q_{t+1};1\right) = A\left(R_t/q_{t+1}\right)$ and $\omega_{t+1}^a = XR_t/\left(\lambda\gamma q_{t+1}\right)$. Using the new notations, the *bona fide solvency constraint* of the bank is

$$D_t \leq (1-\phi)\, \tilde{A}\left(R_t/q_{t+1};\lambda\right) + T_{t+1} = \tilde{A}\left(R_t/q_{t+1};\lambda\right), \tag{38}$$

where $T_{t+1} = \phi\tilde{A}\left(R_t/q_{t+1};\lambda\right)$ represents the required bank capital set aside. The solvency constraint remains broadly unchanged compared to the LF case, except for λ, because the required capital is possessed by the new management of the bank. Although the capital requirement generates the incentives for banks to deleverage, this requirement is much less stringent on the bank's solvency than the bank levy because T_{t+1} is left with banks as usable funds for payout.

In parallel with θ_t^c defined in the previous subsection, a threshold value for the PCA activation needs to be introduced. Suppose that the realized θ_t is larger than a certain level θ_t^a. This condition defines the interest rate and capital price on the brink of the PCA activation such that

$$\frac{1}{1-\phi} D_t = A\left(R_t^a / q_{t+1}^a\right).$$

If the relative price R_t/q_{t+1} exceeds R_t^a/q_{t+1}^a, the PCA is activated. In this case, the banks can remain solvent but are taken into receivership due to undercapitalization. As we define (32) in the BB case, θ_t^a is written as

$$\theta_t^a = \frac{L\left(R_t^a / q_{t+1}^a\right) + w_t}{w_t + D_t + w_{t+1}^a / R_t^a}. \tag{39}$$

By contrast, if $\theta_t > \theta_t^*$, the bona fide solvency constraint (38) is violated and a crisis is precipitated. This condition reintroduces the interest rate and capital price on the brink of financial crises:

$$\frac{1}{1-\phi} D_t = \tilde{A}\left(R_t^* / q_{t+1}^*; \lambda\right) + \frac{1}{1-\phi} T_{t+1}^*. \tag{40}$$

If the relative price R_t/q_{t+1} is greater than R_t^*/q_{t+1}^*, the banking system is precipitated into a crisis, and this is likely to take place for low values of λ. Compared to Problem BL, the bank's solvency is more quickly undermined at the margin because the capital requirement makes leveraging more costly. The capital requirement, however, does not take away the resource that can be used as funds payable to creditors due to the PCA.

In parallel with the practice in previous cases, we define a function R_{PCA}^* as

$$R_{PCA}^*\left(D_t\right) = q_{t+1}^* \tilde{A}^{-1}\left(\frac{D_t}{1-\phi} - \frac{T_{t+1}^*}{1-\phi}; \lambda\right). \tag{41}$$

Note that, alternatively, the inverse function theorem assures $R_{PCA}^{*\prime}\left(D_t\right) = q_{t+1}^* / \left[(1-\phi)\tilde{A}^{*\prime}\right] < 0$, where $\tilde{A}^{*\prime} \equiv \tilde{A}' \left(R_t^*/q_{t+1}^*; \lambda\right)$.

Under the PCA, the threshold level of the liquidity shock for banks' solvency takes a form similar to (13) in the LF economy. We define a function θ^*_{PCA} as

$$\theta^*_{PCA}(D_t) = \frac{\tilde{L}\left(R_t^*/q_{t+1}^*; \lambda\right) + w_t}{w_t + D_t + w_{t+1}^*/R_t^*}, \tag{42}$$

where $\tilde{L}_t = \tilde{L}(R_t/q_{t+1}; \lambda) = \int_{\omega_L}^{\omega_{t+1}^a} Xh(\omega)\, d\omega$ is the liquidity supply under the PCA, which is also a function of λ and $R_t^* = R^*_{PCA}(D_t)$ from (41). The liquidity market clearing condition under the PCA is

$$\tilde{L}(R_t/q_{t+1}; \lambda) = \theta_t \left(\frac{w_{t+1}}{R_t} + D_t\right) - (1-\theta_t)\, w_t. \tag{43}$$

We then state the banks' problem under the capital requirement with the PCA:

Problem PCA *Let $\theta_t^* = \theta^*_{PCA}(D_t)$. In an economy with the capital requirement with the prompt corrective action (PCA), banks maximize*

$$\begin{aligned}
&\max_{D_t} \int_0^{\theta_t^a} \left\{\theta_t \ln(w_t + L_t) + (1-\theta_t)\ln\left[w_{t+1} + R_t(D_t - L_t)\right]\right\} f(\theta_t)\, d\theta_t \\
&+ \int_{\theta_t^a}^{\theta_t^*} \left\{\theta_t \ln\left(w_t + \tilde{L}_t\right) + (1-\theta_t)\ln\left[w_{t+1} + R_t\left(D_t - \tilde{L}_t\right)\right]\right\} f(\theta_t)\, d\theta_t \\
&+ \int_{\theta_t^*}^{1} \left[\theta_t \ln(w_t + X) + (1-\theta_t)\ln(\underline{w})\right] f(\theta_t)\, d\theta_t.
\end{aligned} \tag{44}$$

subject to (9), (43), (39), and (42).

This policy intervention can reduce the crisis probability compared with that in the LF (and BL) economies, as shown in the first column of Table 4. The probability is 6.32 percent when $\lambda = 1$, lower than 6.58 percent in the LF economy. The capital requirement *per se* discourages banks' risk-taking while keeping the banks' resources that are payable to creditors unchanged. This improved resilience of the banking system, however, may not be achieved under alternative conditions. As the third column in Table 4 ($\lambda = 0.75$) indicates, the crisis probability is 7.30 percent, which is higher than that in the LF economy. If the banks under PCA are run by low-skilled bankers, the risk-reduction effect through deleveraging could be dominated by the perils of lower solvency owing to the loss of resources. The overall assessment regarding the capital requirement with the PCA suggests a straightforward point: the success of this policy option largely depends on how efficiently the GC can manage the troubled banks under receivership.

Finally, we reiterate that this result should not be interpreted as either welfare improvement or deterioration. Identifying the optimal policy design requires a full-fledged assessment of policy alternatives with a well-defined social welfare function.

6 Relationship to the Literature

Contrasting our model with the existing literature would crystallize the contribution of this work. The primary aspect of our model is the endogenously varying crisis probability. With this utmost factor in mind, we discuss how our model could be aligned in comparison with the three strands of literature: (i) theories of banking, (ii) macroeconomic models with financial sectors and (iii) models of pecuniary/credit externalities.

6.1 Theories of Banking

Our model is a straightforward extension of Allen and Gale (1998) and Diamond and Rajan (2012) in terms of the basic modeling approach of banks or a banking system. The well-thought microfoundations of banks in those models are essential. A notable feature of the banking systems in those models is that the banks can achieve socially optimal allocation in the absence of government intervention. In fact, Diamond and Rajan (2012) argue that expectations of a sort of bailouts (e.g., protracted low interest policy at a time of crisis) can ill-incentivize banks to be overleveraged. While we agree with Diamond and Rajan (2012), we additionally explore inner sources of inefficiency within banking systems as we detect possibilities that even if there were no expectations for bailout, bank overleverage and resulting financial crises could take place. The fragility of banking systems explored in our work differs from sunspot-driven multiple equilibria, originally suggested by Diamond and Dybvig (1983) and later developed by others. As noted earlier, in our model financial crises are precipitated by fundamental shocks (liquidity preferences) rather than self-fulfilling expectations. We emphasize that financial crises in our model are not unpredictable, entirely random events, but the consequence of excessive risk-taking of banking systems.

6.2 Macroeconomic Models with Banks

Macroeconomic models with financial intermediaries have primarily focused on how financial frictions amplify business cycles (Bernanke and Gertler 1989, Kiyotaki and Moore 1997) rather than how and why devastating financial crises could take place sporadically beyond the business cycle frequency. Nonetheless, a large number of macroeconomic models with "banks" make remarkable progress by extending those state-of-the-art frameworks in interpreting the real economic fluctuations in multiple dimensions.[27] Some recent studies, motivated by the fact that the real economic activities were vastly disrupted by the banking-sector-oriented crisis, discussed how banks with the limited commitment exacerbate economic downturns. On this front, among others, Gertler and Karadi (2011) shed light on the moral hazard problem between financial intermediaries and households.[28] Gertler and Kiyotaki (2011) generalize a similar framework to consider the liquidity management of banks via the interbank market. Gertler, Kiyotaki, and Queralto (2011) extend the Gertler-Kiyotaki model by allowing financial intermediaries to issue both outside equity and short-term debt and argue that lower risks perceived by banks increase bank's leverage and exacerbate economic downturns. They also focus on how unconventional credit policies affect amplification mechanism in the model with banks characterized by the agency problem. While these studies successfully incorporate financial intermediaries or "banks" into the canonical dynamic general stochastic equilibrium models, the business of banking translated in those models still appears somehow over-simplified compared to those handled in the theories of banking as mentioned in the previous subsection.

The focus of our model starkly contrasts with those studies in the following aspects: we aim

[27] Bernanke, Gertler, and Gilchrist (1999), Carlstrom and Fuerst (1997), and Kato (2006) introduce capital mutual funds (CMFs), which merely pool households' funds and lend them to borrowers. As fully discussed in both of those studies, the CMF is an intermediation system that operates fairly mechanically. Using a similar framework, Christiano, Motto, and Rostagno (2010) build a model with a broad spectrum of extensions to answer a number of quantitative questions.

[28] Meh and Moran (2010) and Hirakata, Sudo, and Ueda (2009) adopt two-sided agency problems, by extending Bernanke, Gertler, and Gilchrist's (1999) framework. Nishiyama, Iiboshi, Matsumae, and Namba (2011) also consider a similar two-sided agency problems based on Gertler and Karadi's (2011) framework.

(i) to account for the vast standstill of financial intermediations and subsequent sharp declines in output and investment, both of which the global economy recently experienced, and (ii) to explore a rationale for government intervention based on a solid and explicit welfare assessment. For the first issue, we stress that, while these preceding studies in principle focused on the amplification mechanism within business cycle frequency, our focus is to model systemic financial crises, which are rare but large events beyond business cycle frequency, and subsequent sharp declines in the vast economic activity. For the second, it could be said that our motivation is more fundamental. While those early studies analyze the effect of the credit policies on business cycle fluctuations, we seek for why policy interventions are, if at all, needed. To better understand why we need (or do not need) government interventions in banking systems, we explore why an LF banking system fails to achieve the (constrained) optimal outcome.

Apart from those main differences from the preceding macroeconomic models with banks, we also stress that bank's liquidity shortage in our model plays more prominent roles on bank's insolvency when a financial crisis unfolds. In Gertler and Kiyotaki (2011), for example, the banks are faced with a liquidity constraint but liquidity shortage never leads to bank's insolvency or a financial crisis, even in the case that a large adverse shock erodes the quality of banks' real assets. In this context, Angeloni and Faia (2012) take a step forward by incorporating banks' insolvency into the canonical dynamic general equilibrium model.[29] They build a model based on Diamond and Rajan (2000, 2001a) to consider the macroeconomic consequences of the interaction of banking sector regulations and monetary policy, taking into account the bank's optimal choice of capital structure. Whereas their model has the endogenous probability of bank insolvency, the probability of the bank insolvency in Angeloni and Faia (2012) can broadly be interpreted as a measure of individual bank fragility rather than the probability of financial crises where the vast majority of the banking system comes to a standstill. Our model assesses macroeconomic fragility in terms of the probabilities of a sporadic financial crisis rather than perpetually changing fraction of insolvent banks in the system. We also highlight the non state-contingency of banks' liabilities not only be-

[29] In a similar context, Kobayashi and Nakajima (2011) incorporate a banking system developed by Diamond and Rajan (2001a) into the infinite-horizon DSGE model in an attempt to investigate how systemic bank runs amplify recessions.

cause it is essential for the unique business of banking compared to other financial intermediaries, but because of an additional reason: The non state-contingency is a key ingredient of our model as it creates the venue where pecuniary externalities arise. We will discuss this issue in the next subsection.

6.3 Pecuniary Externalities

In the wake of the insolvency of the Lehman Brothers in September 2008 and amid the ensuing repercussions, a big question mark was posed on the status quo banking systems. Against the backdrop, we address whether financial crises could be understood as an inefficient outcome arising from over risk-taking of the banking sector. A number of similar attempts have been made to examine why and how over-credit/over-borrowing can arise under LF economies with the aim to better focus on tail risks (i.e., financial crises) in a general equilibrium context.

Lorenzoni (2008) set out a formal model where an LF economy tends to result in over-borrowing and suggests that inefficient credit booms could be a natural outcome of competitive financial transactions. Prompted by Lorenzoni's (2008) thought-provoking and generally applicable framework, a number of studies explore the source of over-credit in an attempt to reaffirm the microfoundation of government interventions, including macro-prudential regulations. In this line of work, Bianchi (2010), Bianchi and Mendoza (2010), Korinek (2010) and Jeanne and Korinek (2012a, b), Mendoza (2010), among others, have reached a broad consensus that pecuniary externalities, combined with some sorts of incomplete market or/and limited commitment, can fairly in general prevent LF markets from achieving socially optimal outcomes.[30] Many of these models assume that borrowers are constrained by collateral constraints and the constraints depend on some prices (e.g., capital prices) that the agents take as given. Owing to the pecuniary externalities, the price-taking behaviors do not achieve the constrained optimum. Our model follows their idea in terms of pecuniary externalities that create inefficiency in financial intermediation.

We agree with Lorenzoni and others on the basic idea that the pecuniary externalities are likely

[30] Bengui (2011) incorporates multiple debts with different maturities into a similar framework to the aforementioned models. As a result, he finds that in the LF economy, short-term debt is overissued and the fundamental shocks propagate more powerfully.

to perform the key roles in creating inefficient boom-bust cycles. However, we note that overemphasis on borrowing constraints (or collateral constraint) may sometimes be debatable because evidently some cash-rich investors could be free from such constraints. A marked contrast among our model and the other early studies is that, in our model, pecuniary externalities operate on the banks' solvency constraints rather than borrowing constraints of entrepreneurs or other non-bank agents. As far as banks that issue non-state-contingent debt are acting in place, they are all faced with solvency constraints which naturally depend on some prices of financial assets.

In this regard, Stein (2012) considers pecuniary externalities affecting the business of "banks" and shows the possibilities that the banks would overissue short-term debt. In his model, the maximum amount of banks' short-term debt is determined by the fire-sale price of their own assets. He provides a well-designed framework and successfully discusses overleverage in financial system and its remedial measures. While we assess the probability of inefficient financial crises by explicitly modeling banks that are subject to risks of systemic bank runs, he focuses more on how to contain risks of fire sales. In Stein (2012), short-term creditors are always secure because banks do not lever up beyond the borrowing constraint. Thus, although systemic bank runs are the zero probability event in his model, inefficiency due to pecuniary externalities arises in the form of fire sales and socially excessive money creation.

In sum, our view is that, while the pecuniary externalities remain as the key factor, detecting where they perform in financial system matters. In light of our main purpose, incorporating pecuniary externalities into banking system itself à la Diamond and Rajan (2001a, 2012) appears to be a sensible approach to model systemic financial crises.[31]

7 Conclusions

We developed a dynamic general equilibrium model that explicitly includes banks with a maturity mismatch. Using the model, we showed that, under the laissez-faire economy, inefficient financial

[31] Uribe (2011) argues that as long as the borrowing constraint with a chosen form is affected by asset prices, pecuniary externalities arise in the models while — depending on the specification of the borrowing constraint (e.g., how it would depend on borrowers' collateral) — the results of over- or under-leverage can vary. Compared with the borrowing constraints, which can be applied in a variety of forms, banks' solvency constraint may be given less leeway and, accordingly, the results for the debt-issuing banks may hold more generally.

crises are precipitated by a liquidity shortage in the overleveraged banking system. In general, a competitive banking sector cannot achieve the first-best allocation because the banking business *per se* (i.e., maturity transformation via issuance of non-state-contingent debt rather than Arrow securities) implies that financial markets are incomplete.

Our model demonstrated that a competitive banking sector cannot always achieve the second-best allocation, resulting in an inefficiently high exposure to crisis risks. In our model, pecuniary externalities arise, distorting the individual banks' assessment of the systemic risks. The banks fail to internalize the side effects of changes in the illiquid asset prices on their own solvency. From the viewpoint of a social planner, because of this failure, the individual banks overestimate their solvency of the banking system as a whole. This pecuniary externality exposes banks to inefficiently elevated systemic risks. In the light of real-world experience, our model could serve as a foundation for a better understanding of the repeatedly observed financial and economic crises. We also assessed the three policy options — (i) bank levy, (ii) bank bailout, and (iii) capital requirement with prompt corrective action — in an attempt to reduce inefficient crisis risks. Among others, the results point to the possible perils of an anticipated bank bailout and commitment to a low interest rate policy, because these options would ill-incentivize banks to take on even higher risks by undermining discipline in the banking system.

The analysis demonstrated in this paper can be extended in a number of directions. First, it may be necessary to examine how changes in a variety of economic environments (e.g., changes in the stochastic process of the liquidity shock) or newly introduced aggregate shocks (e.g., shocks to the asset side of banks' balance sheets) affect the economy's exposure to crisis risks and macroeconomic fluctuations.[32] Second, introducing price stickiness into the model would pave the way for reconstruction of the roles of monetary policy in comparison with similar models that do not include the possibility of a financial crisis. All of these directions would provide important steps for future research.

[32] Kato and Tsuruga (2011) demonstrate that rational banks can take on more risks, resulting in a higher default probability, in response to changes in the underlying distribution of shocks that reduce exogenous risks of bank defaults.

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A Appendix

A.1 Proof of Proposition 1

We first show Lemma 1 which indicates that $dR_t^*/dD_t < 0$ and then proves the proposition.

Lemma A.1 $dR_t^*/dD_t < 0$**.**

Proof By taking total derivatives of (18) with respect to D_t, we obtain

$$\frac{dR_t^*}{dD_t} = \left(\frac{1}{1 - F_{KK,t+1}^* \Phi^{*\prime} R_t^*/q_{t+1}^*} \right) \frac{q_{t+1}^*}{A^{*\prime}}.$$

Then it suffices to show $F_{KK,t+1}^* \Phi^{*\prime} R_t^*/q_{t+1}^* < 1$. Applying the implicit function theorem to (22) yields

$$\Phi^{*\prime} = \frac{1}{q_{t+1}^*/I^{*\prime} + R_t^* F_{KK,t+1}^*/q_{t+1}^*},$$

where $I^{*\prime} = I' \left(R_t^*/q_{t+1}^* \right) < 0$. Hence,

$$F_{KK,t+1}^* \Phi^{*\prime} \frac{R_t^*}{q_{t+1}^*} = \frac{1}{1 + \left(q_{t+1}^* \right)^2 / \left(R_t^* I^{*\prime} F_{KK,t+1}^* \right)} < 1,$$

which proves Lemma A.1. ■

Recall that the discrepancy between $R_{LF}^{*\prime} \left(D_{LF,t} \right)$ and $R_{SP}^{*\prime} \left(D_{LF,t} \right)$ is given by (27)

$$R_{LF}^{*\prime} \left(D_{LF,t} \right) - R_{SP}^{*\prime} \left(D_{LF,t} \right) = -\frac{R_t^*}{q_{t+1}^*} F_{KK,t+1}^* \Phi^{*\prime} \frac{dR_t^*}{dD_t} > 0.$$

The sign of the discrepancy is assured by Lemma A.1.

A.2 Efficiency Condition in the Case of a Bank Bailout

To clarify differences in the efficiency condition in the BB economy from that in the BL economy, we write out the efficiency condition under the BL economy:

$$\begin{aligned}
&\left\{\theta_t^* \log\left(\frac{\theta_t^* m_t^*}{w_t + X}\right) + (1-\theta_t^*)\log\left[\frac{(1-\theta_t^*) R_t^* m_t^*}{\underline{w}}\right]\right\} \frac{d\pi_t}{d\theta_t^*}\theta_{BL}^{*\prime}(D_t) \\
= & \int_0^{\theta_t^*}\left[\frac{1-\left(w_{t+1}/R_t^2\right) R_{BL}'(D_t,\theta_t)}{m_t} + \frac{(1-\theta_t) R_{BL}'(D_t,\theta_t)}{R_t}\right] f(\theta_t)\, d\theta_t, \qquad (45)
\end{aligned}$$

where $\theta_{BL}^{*\prime}(D_t)$ and $R_{BL}^{*\prime}(D_t)$ are given by

$$\begin{aligned}
\theta_{BL}^{*\prime}(D_t) &= \frac{1}{m_t^*}\left[\frac{\partial}{\partial R_t^*}(L_t^* - g_t^*)\right] R_{BL}^{*\prime}(D_t) - \frac{\theta_t^*}{m_t^*} &(46)\\
R_{BL}^{*\prime}(D_t) &= \frac{(1+\tau)\, q_{t+1}^*}{A^{*\prime}} &(47)\\
R_{BL}'(D_t,\theta_t) &= R_{LF}'(D_t,\theta_t).
\end{aligned}$$

Note that $\theta_{BL}^{*\prime}(D_t)$ and $R_{BL}^{*\prime}(D_t)$ reflect the price-taking banks' behavior that take factor prices as given. Here, compared to the efficiency condition under the LF economy, the efficiency condition (45) remains the same except that the levy τ affects the marginal change in the crisis probability. The bank levy affects banks' risk-taking through $R_{BL}^{*\prime}(D_t)$ in (47) because banks recognize that their own solvency is undermined by the levy and they are faced with higher risks of insolvency. Given any level of D_t and θ_t, $R_{BL}'(D_t,\theta_t)$ remains the same as $R_{LF}'(D_t,\theta_t)$ defined in (17) because, unless a crisis takes place, the banks' leverage D_t influences R_t under the BL economy in the same way as under the LF economy.

In contrast, under the BB economy, the following efficiency condition can be obtained:

$$\begin{aligned}
&\left\{\theta_t^* \log\left(\frac{\theta_t^* m_t^*}{w_t + X}\right) + (1-\theta_t^*)\log\left[\frac{(1-\theta_t^*) R_t^* m_t^*}{\underline{w}}\right]\right\} \frac{d\pi_t}{d\theta_t^*}\theta_{BB}^{*\prime}(D_t) \\
= & \int_0^{\theta_t^c}\left[\frac{1-\left(w_{t+1}/R_t^2\right) R_{BB}'(D_t,\theta_t)}{m_t} + \frac{(1-\theta_t) R_{BB}'(D_t,\theta_t)}{R_t}\right] f(\theta_t)\, d\theta_t \\
& + \int_{\theta_t^c}^{\theta_t^*}\left[\frac{1-\left(w_{t+1}^*/R_t^{*2}\right) R_{BB}^{*\prime}(D_t,\tau)}{m_t^*} + \frac{(1-\theta_t) R_{BB}^{*\prime}(D_t)}{R_t^*}\right] f(\theta_t)\, d\theta_t, \qquad (48)
\end{aligned}$$

where $\theta^{*\prime}_{BB}(D_t)$ and $R^{*\prime}_{BB}(D_t)$ are defined as

$$\theta^{*\prime}_{BB}(D_t) = \frac{1}{m^*_t}\left[\frac{\partial}{\partial R^*_t}(L^*_t - g^*_t)\right] R^{*\prime}_{BB}(D_t) - \frac{\theta^*_t - \tau}{m^*_t} \quad (49)$$

$$R^{*\prime}_{BB}(D_t) = R^{*\prime}_{BL}(D_t) \quad (50)$$

$$R'_{BB}(D_t, \theta_t) = R'_{BL}(D_t, \theta_t) = R'_{LF}(D_t, \theta_t).$$

Because the GC continues to impose the levy on banks, $R^{*\prime}_{BB}$ remains the same as $R^{*\prime}_{BL}$. As noted, $R'_{BB}(D_t, \theta_t)$ remains the same as $R'_{LF}(D_t, \theta_t)$ and $R'_{BL}(D_t, \theta_t)$.

Equation (48) represents the marginal cost and benefit on each side. The marginal cost on the left-hand side appears to be the same as the marginal cost under the BL economy. In the case of a bank bailout, however, the risks of insolvency are mitigated by the GC's emergency liquidity provision as the intervention reins in the interest rate, propping up the banking system. In terms of equations, the lower risks can be confirmed by comparisons between the second terms of (46) and (49). In particular, given D_t, $\theta^{*\prime}_{BB}(D_t)$ is assured to be smaller than $\theta^{*\prime}_{BL}(D_t)$ (in the absolute value) by Proposition 3.

The marginal benefit on the right-hand side reflects the GC's commitment to the low interest rate policy. In parallel with the marginal cost, the first term is similar to the right-hand side of (45), because, for the lowest range of $\theta_t \in [0, \theta^c_t]$, the GC does not step into the liquidity market and plays no role in the marginal benefit of increasing leverage. For the middle range of $\theta_t \in (\theta^c_t, \theta^*_t)$, however, the GC intervenes in the liquidity market to supply the emergency liquidity and households are faced with the market interest rate at the pre-committed level R^*_t. For this reason, the second term on the right-hand side of (48), which is the marginal benefit of increasing leverage for $\theta_t \in (\theta^c_t, \theta^*_t)$, includes $R^{*\prime}_{BB}(D_t)$ rather than $R'_{BB}(D_t, \theta_t)$.

Table 1: Sequence of events for generation t

	Period t
1.	Households receive endowments.
2.	Banks offer deposits to households and loans to entrepreneurs.
3.	Entrepreneurs launch their projects.
4.	Households supply labor and receive wages w_t determined by the labor market conditions along with the old generation's labor supply.
5.	Liquidity shock θ_t is realized, and banks receive signals of project outcomes.
6.	Households decide the withdrawal amount g_t.
7.	Banks decide which projects to discontinue and supply liquidity L_t.
(i)	If $g_t > L_t$, a financial crisis is precipitated and households receive repayment of X.
(ii)	Otherwise, the households can transfer their wealth into the period $t+1$.
8.	All agents consume.
	Period $t+1$
1.	Entrepreneurs receive endowments.
2.	Entrepreneurs' projects are completed, and they sell their capital goods for q_{t+1} and make repayment to banks.
3.	Households fully withdraw deposits, if any.
4.	Households supply labor and receive wages w_{t+1} determined by the labor market conditions along with the young generation's labor supply.
5.	All agents consume.

Table 2: Crisis probabilities and allocations

under laissez-faire banking sector and social planning banks

	SP banks	LF banks
Leverage and crisis probabilities		
D_t	1.049	1.061
π_t (%)	4.499	6.585
MSR	1.993	1.544
Bank capital and output		
Bank capital ratio (%)	15.097	13.952
Y_{t+1}	5.459	5.457

Note: Simulation results based on the assumption that the liquidity shock θ_t follows the beta distribution. The level of banks' leverage D_t and the probability of a financial crisis π_t are obtained from Problems LF and SP, respectively. The marginal systemic risk, MSR, is given by (25). The bank capital ratio is $(A_t - D_t)/A_t$.

Table 3: Bank levy and bank bailout

	Bank levy	Bank bailout
A: $\tau = 0.01$		
Leverage and probabilities		
D_t	1.052	1.055
π_t (%)	6.670	6.740
MSR	1.704	1.613
Bank capital and GDP		
Bank capital (%)	13.948	13.660
Y_{t+1}	5.458	5.458
B: $\tau = 0.02$		
Leverage and probabilities		
D_t	1.044	1.050
π_t (%)	6.755	6.890
MSR	1.855	1.673
Bank capital and GDP		
Bank capital (%)	13.943	13.365
Y_{t+1}	5.460	5.459
C: $\tau = 0.03$		
Leverage and probabilities		
D_t	1.036	1.044
π_t (%)	6.840	7.034
MSR	1.996	1.725
Bank capital and GDP		
Bank capital (%)	13.937	13.068
Y_{t+1}	5.462	5.460

Note: The banks' leverage D_t and the probability of a financial crisis π_t are obtained from different values of τ in (28). Panels A, B, and C correspond to the case of $\tau = 0.01$, 0.02, and 0.03, respectively The bank capital ratio excludes the surcharge defined as $[A_t/(1+\tau) - D_t]/[A_t/(1+\tau)]$. The other details can be seen in the note for Table 2.

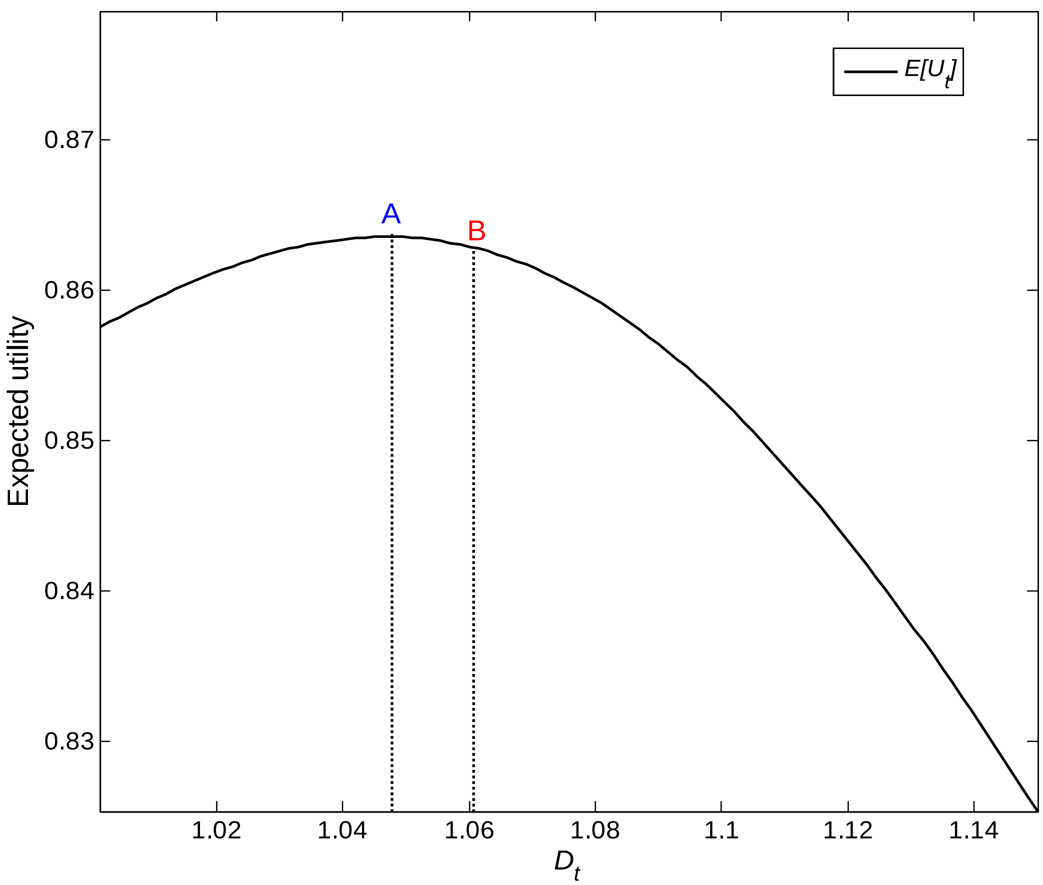
Table 4: Capital requirement with prompt corrective action

A: $\phi = 0.04$			
	$\lambda = 1.00$	$\lambda = 0.95$	$\lambda = 0.75$
D_t	1.060	1.057	1.043
π_t (%)	6.321	6.457	7.298
Y_{t+1}	5.457	5.457	5.460

B: $\phi = 0.06$			
	$\lambda = 1.00$	$\lambda = 0.95$	$\lambda = 0.75$
D_t	1.059	1.056	1.042
π_t (%)	6.188	6.321	7.066
Y_{t+1}	5.457	5.458	5.461

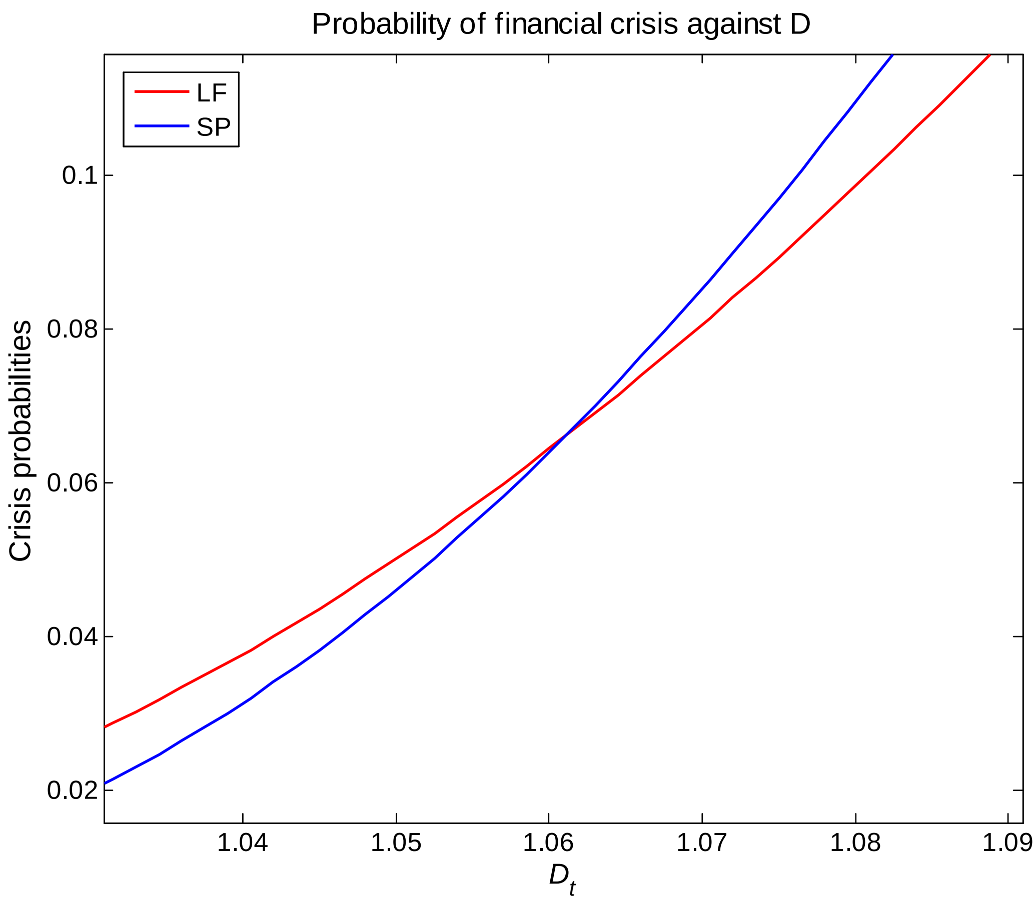
Note: Numbers in the table are obtained under the capital requirement with prompt corrective action across various capital requirement ratios (ϕ) and various degrees of reduction in the value of investment projects (λ) arising from the new management taken by the government/central bank. Panels A and B correspond to the cases of $\phi = 0.04$ and 0.06. Each column shows the degrees of reduction in the value of investment projects (λ). The level of output Y_{t+1} is obtained under the assumption that θ_t takes the mean of 0.5.

Figure 1: Optimal bank leverage



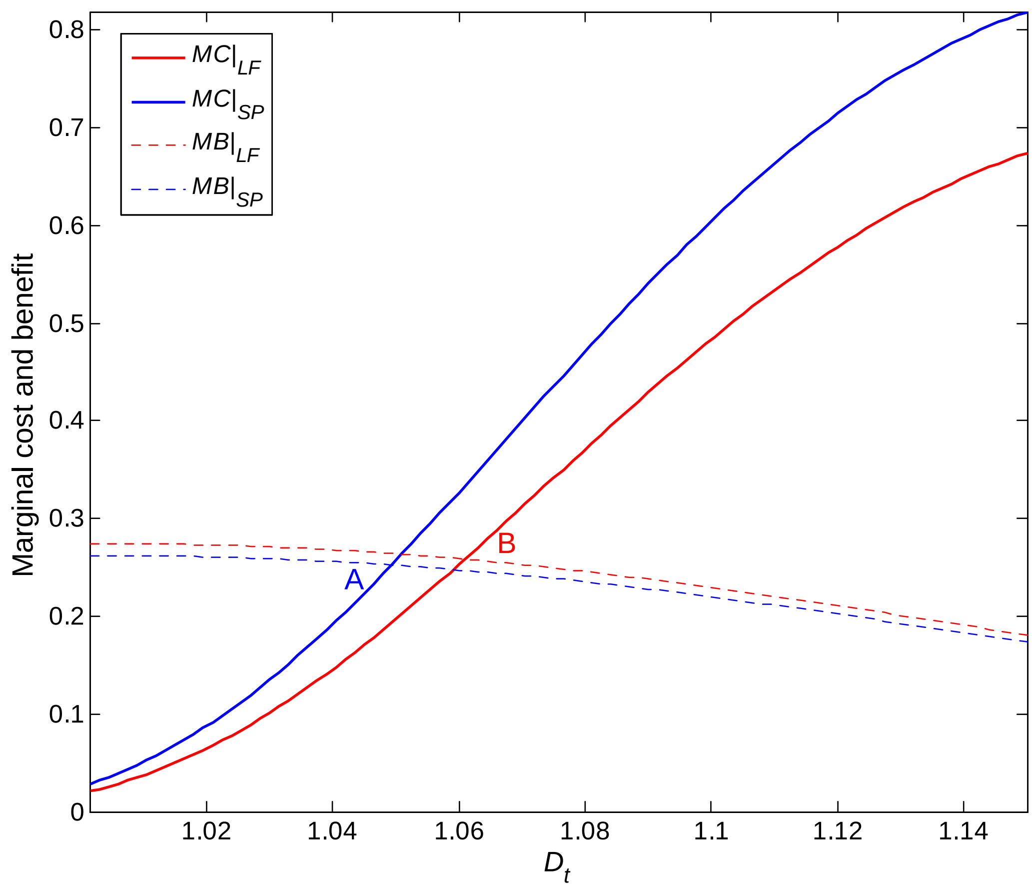
Note: The expected utility against D_t. Point A corresponds to the expected utility level evaluated at D_t chosen by social planning banks, while point B corresponds to the expected utility evaluated at D_t chosen by laissez-faire banks.

Figure 2: Marginal systemic risk (MSR)



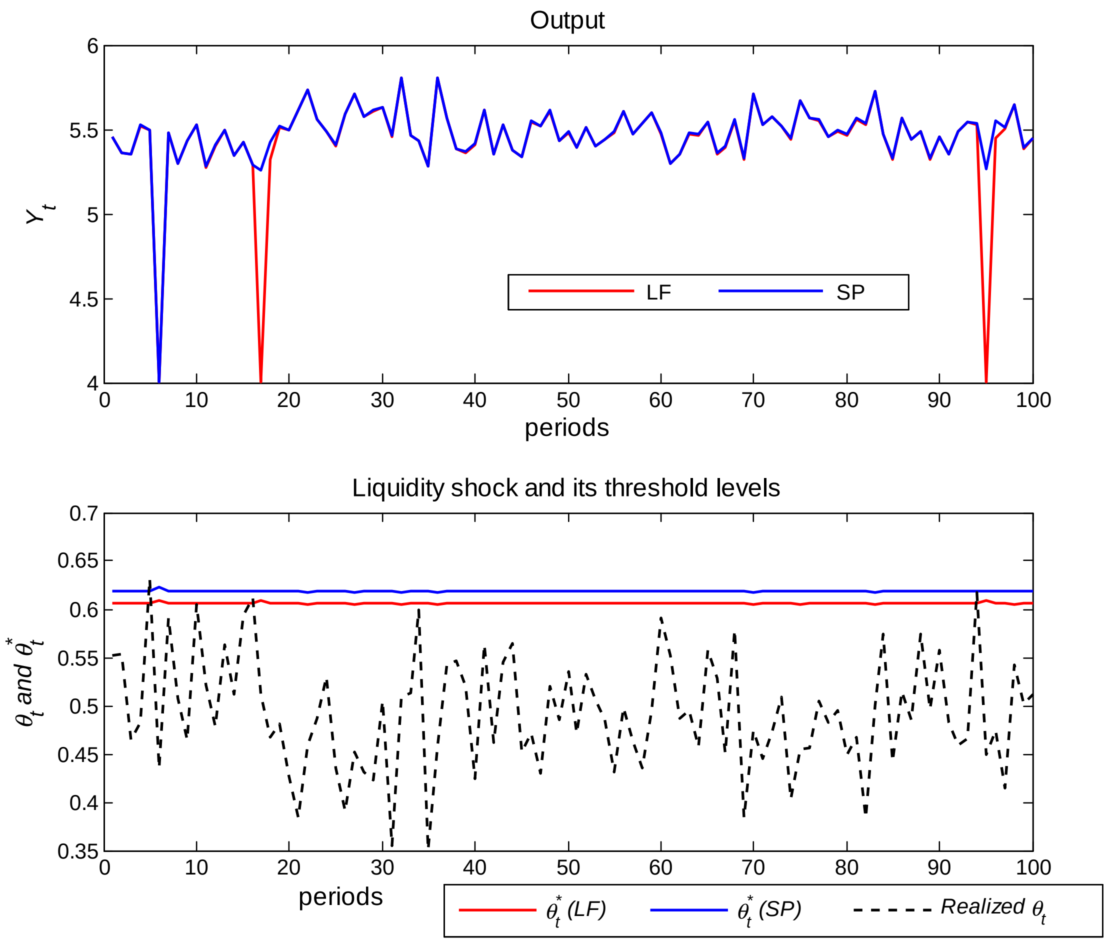
Note: The blue line plots the crisis probability in the general equilibrium allocation. The red line plots the crisis probability that laissez-faire (LF) banks recognize. The slopes of the lines indicate the MSRs under the LF banks and social planning (SP) banks.

Figure 3: Marginal cost and benefit of increasing D_t



Note: The solid blue line ($MC|_{SP}$) and dashed blue line ($MB|_{SP}$) represent the marginal cost and benefit under the social optimum, respectively. The solid red line ($MC|_{LF}$) and dashed red line ($MB|_{LF}$) are the marginal cost and benefit under the laissez-faire economy, respectively.

Figure 4: Simulated paths of output and the liquidity shock



Note: In both panels, the blue lines are for the economy with the social planning (SP) banks and the red lines are for the economy with the laissez-faire (LF) banks. The upper panel shows the simulated dynamic paths of output Y_t. The liquidity shock plotted as the dashed black line in the lower panel is generated from the beta distribution with a mean of 0.50 and a standard deviation of 0.07. The solid blue and red lines in the lower panel are the threshold level of the liquidity shock that satisfies the solvency constraint with equality.