# Conditional probabilities for euro area sovereign default risk

Andre Lucas, Bernd Schwaab, Xin Zhang

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email: bernd.schwaab@ecb.int
webpage: www.berndschwaab.eu

Disclaimer: Not necessarily the views of ECB or ESCB.

# Contributions

We propose a **novel modeling framework** to infer **conditional** and **joint probabilities** for sovereign default risk from observed CDS.

**Novel framework?** Based on a *dynamic GH skewed–t* multivariate density/copula with time-varying volatility and correlations.

Multivariate model is sufficiently flexible to be **calibrated daily** to credit market expectations. Not an "official opinion".

Analysis is based on **Euro area** CDS data from Jan 2008 to June 2011. **Event study**: SMP/EFSF announcement & initial impact on risk.

# Literature

1. **Sovereign credit risk:** e.g. Pan and Singleton (2008), Longstaff, Pan, Pedersen, and Singleton (2011), Ang and Longstaff (2011).
2. **Contagion**, see e.g. Forbes and Rigobon (2002), Caporin, Pelizzon, Ravazzolo, Rigobon (2012).
3. **Observation-driven time-varying parameter models**, see Creal, Koopman, and Lucas (2011, 2012), Zhang, Creal, Koopman, Lucas (2011), Creal, Schwaab, Koopman, Lucas (2011), Harvey (2012).
4. **Non-Gaussian dependence/copula/credit modeling**, see e.g. Demarta and McNeil (2005), Patton and Oh (2011).

# Empirical questions

(**Q1**) Financial stability information: Based on credit market expectations, what is ...

Pr(two or more credit events in Euro area)?

Pr(i|j)-Pr(i), for any i,j?

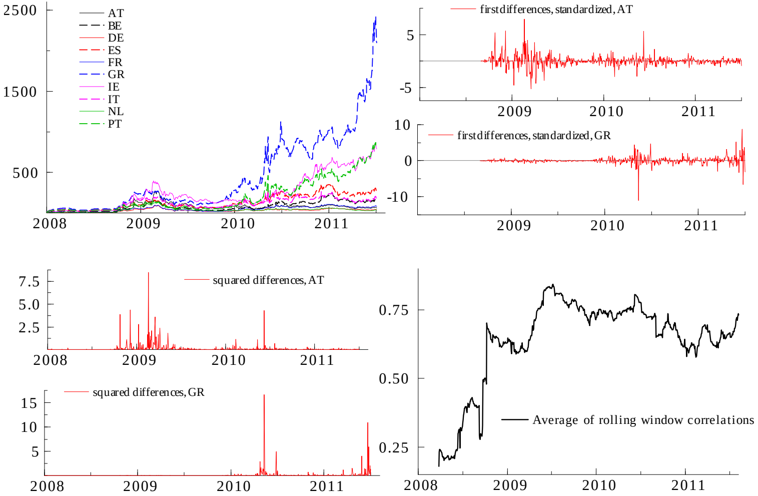
Spillovers, e.g. Pr(PT|GR) - Pr(PT|not GR)?

$\text{Corr}_t$(i,j) at time t?

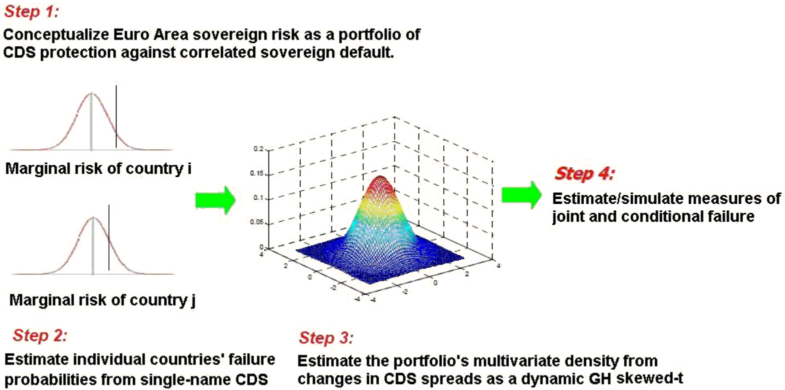
(**Q2**) Model risk: For answering (a), how important are parametric assumptions? *Normal* vs *Student-t* vs *GH skewed-t*.

(**Q3**) Event study: did the May 09, 2010 Euro area rescue package change risk dependence? How?

# Data: skewed, fat tailed, tv vol's and correlation



# The copula idea/road-map



# Copula framework

Sovereign defaults iff benefits $(v_{it})$ exceed a cost $(c_{it})$, where

$$v_{it} = (\varsigma_t - \mu_\varsigma)\tilde{L}_{it}\gamma + \sqrt{\varsigma_t}\tilde{L}_{it}\epsilon_t, \quad i = 1, ..., n,$$

$\epsilon_t \sim$N$(0, I_n)$ is a vector of risk factors,
$\tilde{L}_{it}$ contains risk factor loadings,
$\gamma \in \mathbb{R}^n$ determines skewness,
$\varsigma_t \sim IG$ is an additional scalar risk factor for, say, *interconnectedness*.

A default occurs with probability $p_{it}$, where

$$p_{it} = \Pr[v_{it} > c_{it}] = 1 - F_i(c_{it}) \;\Leftrightarrow c_{it} = F_i^{-1}(1 - p_{it}),$$

where $F_i$ is the CDF of $v_{it}$.

Focus on *conditional* probability $\Pr[v_{it} > c_{it} | v_{jt} > c_{jt}]$, $i \neq j$.

# GH skewed-t dependence

$$y_t = \mu + L_t e_t, \quad t = 1, ..., T, \; e_t \sim \text{GHST}, \; \text{E}[e_t e_t'] = I_n,$$

$$p(y_t; \cdot) = \frac{\upsilon^{\frac{\upsilon}{2}} 2^{1-\frac{\upsilon+n}{2}}}{\Gamma\left(\frac{\upsilon}{2}\right) \pi^{\frac{n}{2}} \left|\tilde{\Sigma}_t\right|^{\frac{1}{2}}} \cdot \frac{K_{\frac{\upsilon+n}{2}}\left(\sqrt{d(y_t)\cdot(\gamma'\gamma)}\right) e^{\gamma' \tilde{L}_t^{-1}(y_t - \tilde{\mu}_t)}}{\left(d(y_t)\cdot(\gamma'\gamma)\right)^{-\frac{\upsilon+n}{4}} d(y_t)^{\frac{\upsilon+n}{2}}},$$

where

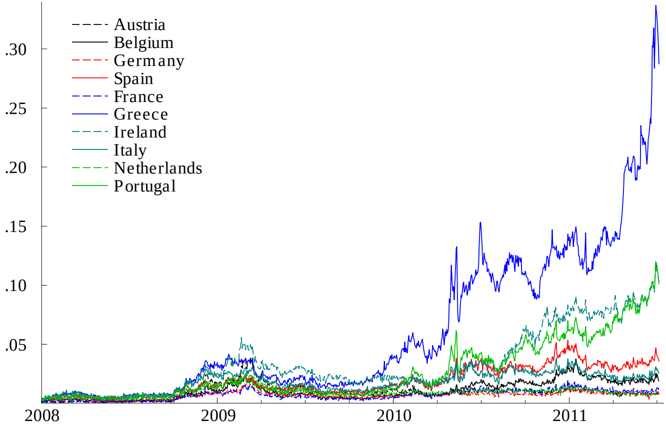
$$d(y_t) = \upsilon + (y_t - \tilde{\mu}_t)' \tilde{\Sigma}_t^{-1} (y_t - \tilde{\mu}_t),$$

$$\tilde{\mu}_t = -\upsilon/(\upsilon-2)\; \tilde{L}_t \gamma,$$

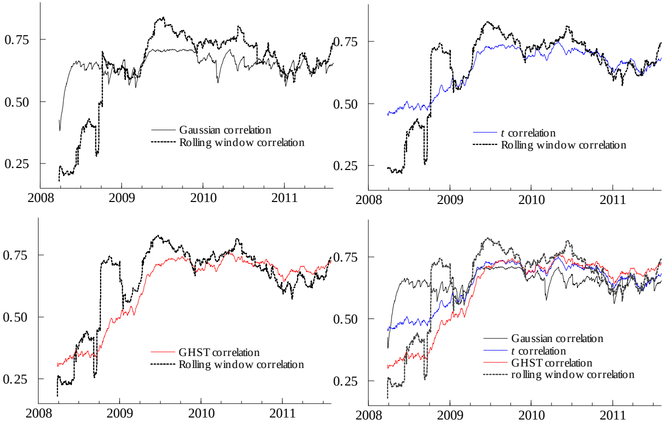
$$\tilde{\Sigma}_t = \tilde{L}_t \tilde{L}_t' \quad \text{is scale matrix}$$

If $\gamma = 0$, then GH skewed-$t$ simplifies to Student's $t$ density.
If in addition $\upsilon^{-1} \to 0$, then multivariate Gaussian density.
$\tilde{\Sigma}_t(f_t) = \tilde{L}_t(f_t)\tilde{L}_t(f_t)'$ is driven by 1st and 2nd derivative of the pdf.

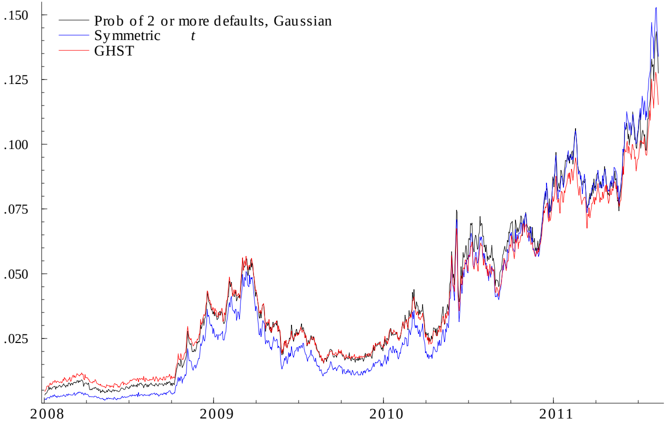
# Marginal pd's from CDS



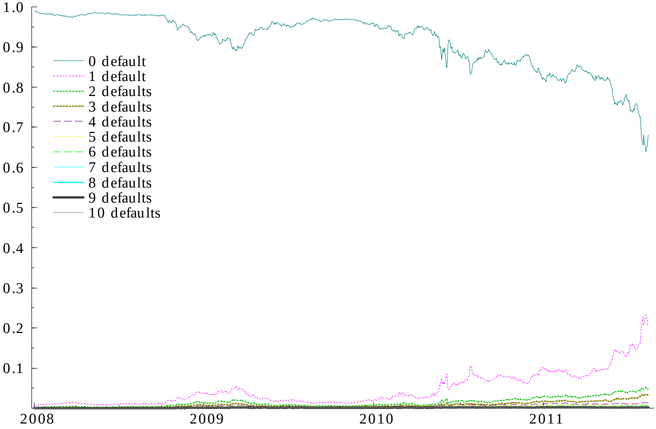
# Average correlation



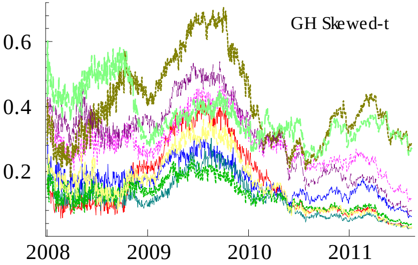
# The probability of two or more credit events

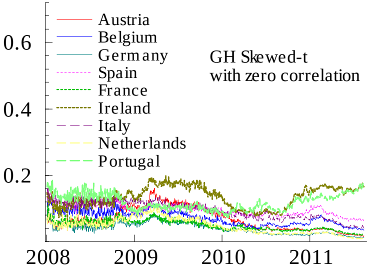


# The probability of k=0,1,2,... failures

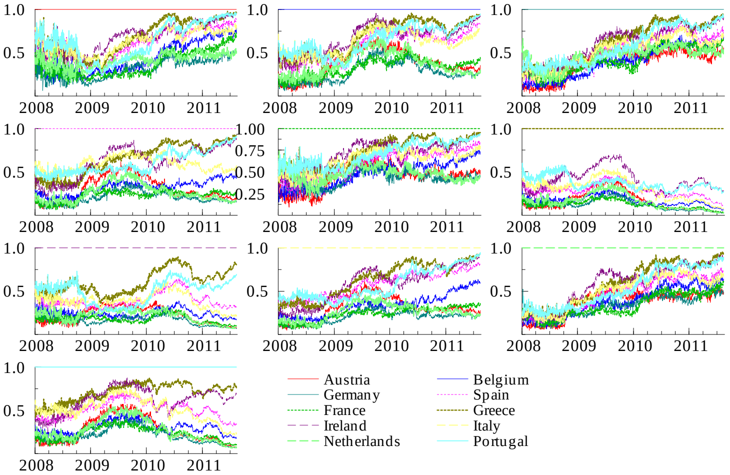


# Conditional pds: Pr(country i|GR)



# Conditional pds: Pr(all i| all j)



# The May 09, 2010 package

| Joint risk, $\Pr(i \bigcap j)$ | | | | | | |
|---|---|---|---|---|---|---|
| | Thu 06 May 2010 | | | Tue 11 May 2010 | | |
| | PT | GR | DE | PT | GR | DE |
| AT | 1.1% | 1.1% | 0.6% | 0.6% | 0.7% | 0.4% |
| BE | 1.2% | 1.4% | 0.7% | 0.9% | 1.0% | 0.6% |
| DE | 1.0% | 1.1% | | 0.8% | 0.8% | |
| ES | 3.0% | 3.3% | 0.9% | 1.5% | 1.6% | 0.6% |
| FR | 1.0% | 1.0% | 0.6% | 0.8% | 0.9% | 0.6% |
| GR | **4.8%** | | 1.1% | **2.3%** | | 0.8% |
| IR | **2.6%** | **3.1%** | 0.8% | **1.4%** | **1.8%** | 0.6% |
| IT | 2.8% | 2.9% | 0.9% | 1.4% | 1.5% | 0.6% |
| NL | 0.9% | 0.9% | 0.5% | 0.6% | 0.7% | 0.5% |
| PT | | 4.8% | 1.0% | | 2.3% | 0.8% |
| Avg | **2.0%** | **2.2%** | 0.8% | **1.1%** | **1.2%** | 0.6% |

# The May 09, 2010 package

| Conditional risk, Pr(i \| j) | | | | | | |
|---|---|---|---|---|---|---|
| | Thu 06 May 2010 | | | Tue 11 May 2010 | | |
| | PT | GR | DE | PT | GR | DE |
| AT | 17% | 8% | 53% | 22% | 10% | 46% |
| BE | 20% | 10% | 60% | 32% | 15% | 61% |
| DE | 16% | 8% | | 26% | 12% | |
| ES | 49% | 25% | 78% | 50% | 23% | 63% |
| FR | 16% | 8% | 58% | 28% | 12% | 62% |
| GR | **78%** | | 99% | **80%** | | 86% |
| IR | **43%** | **23%** | 75% | **49%** | **26%** | 68% |
| IT | 45% | 22% | 77% | 49% | 21% | 64% |
| NL | 14% | 7% | 49% | 21% | 10% | 50% |
| PT | | 36% | 91% | | 33% | 81% |
| Avg | **33%** | **16%** | 71% | **40%** | **18%** | 64% |

Bottom line: joint risks ↓↓, but dependence ↑. "Firewall"-analogy?

# Conclusion

We propose a **novel modeling framework** to infer **conditional** and **joint probabilities** for sovereign default risk from observed CDS.

**Novel framework?** Based on a *dynamic GH skewed–t* multivariate density/copula with time-varying volatility and correlations.

# Thank you