Debt Sustainability in a Low Interest Rate World

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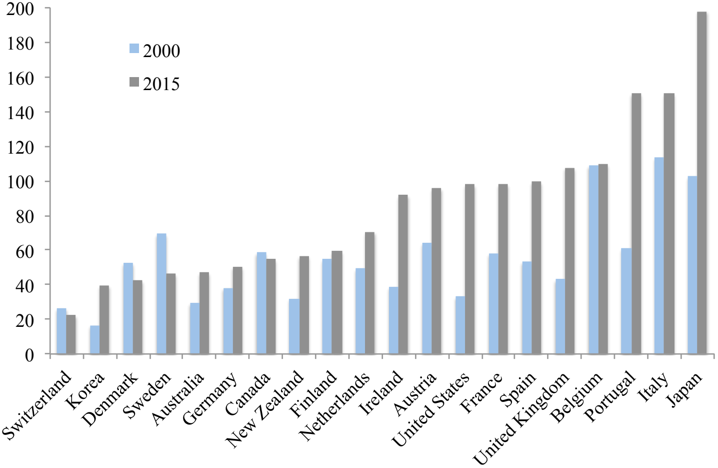
PUBLIC DEBT

OECD ECONOMIES



DEBT SERVICING COSTS

OECD ECONOMIES

Research Question and Approach:

Key tradeoff:

- Persistent $r < g$ allows for larger sustainable primary deficits
- With a large stock of public debt, interest rate reversals can impose sizable fiscal costs
- Weak growth has counteracting effects on debt dynamics

Approach:

- Empirical evidence on historical level and variability of $r - g$
- Utilize a continuous time model to study implications for debt servicing cost of "secular stagnation" scenarios

Preview of Findings

Empirical findings:

- Average cost of servicing the public debt is close to zero
- Substantial variability and reversion risk in $r - g$

Analytical findings:

- Possibility of stationary debt to GDP absent any fiscal response
- Slower productivity growth may *improve* debt sustainability
- Elevated risk premia carry ambiguous effects for debt dynamics
- Findings carry over to an environment with default

Outline for Presentation

1. **Empirical facts**

2. Case of no default

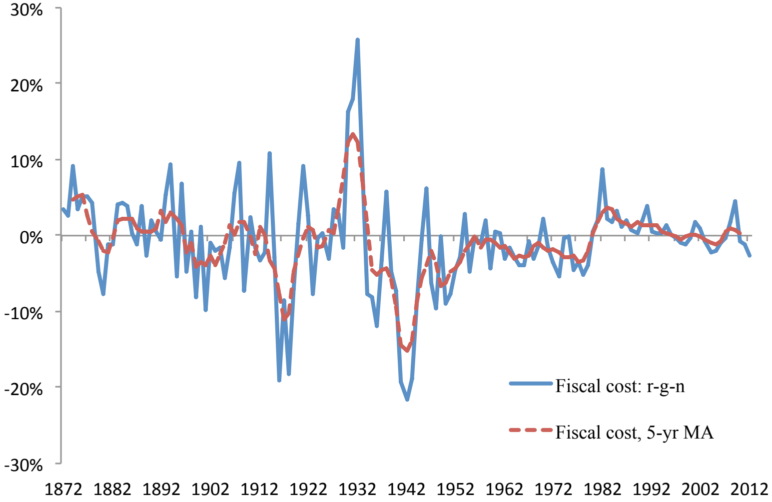
3. Case of default

Historical debt servicing cost

	17 Advanced Countries		United States	
	1870-2013	1946-2013	1870-2013	1946-2013
Net fiscal cost: r - (g+n)				
25th percentile	-2.64	-2.74	-2.15	-1.72
Median	0.08	-0.38	-0.16	-1.35
75th percentile	2.28	1.55	1.09	0.57
Fraction < 0	49.3%	54.3%	55.2%	69.2%
Fraction < -2%	31.4%	32.6%	31.0%	23.1%
No. of observations	493	221	29	13

- Median cost of servicing the debt is close to zero for all economies
- Significant fraction of time with cost of servicing the debt very negative

Cost of servicing the US public debt



Outline for Presentation

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Economic Environment

- Time: $t \geq 0$
- Goods: consumption
- Agents: representative household, fiscal authority
- Assets: risky capital, government bonds
- Uncertainty: endowment, fiscal policy

$$dY_t = gY_t dt + \sigma_y Y_t dZ_t^y$$

Households

Objective and constraints

$$\max_{c_t,a_t,x_t,b_t,s_t} W_t = V_t + \mathbb{E}_t \int_t^\infty \pi_{t,s} Y_s u\left(\frac{b_s}{Y_s}\right) ds$$

$$V_t = \mathbb{E}_t \int_t^\infty f(c_s, V_s) ds$$

$$f(c_s, V_s) = \frac{((1-\gamma)V_s)^{\frac{\theta-\gamma}{1-\gamma}}}{1-\theta} \left[c_s^{1-\theta} - (\rho - n)\left((1-\gamma)V_s\right)^{\frac{1-\theta}{1-\gamma}} \right]$$

$$\text{s.t.: } da_t = (r_t^s s_t + r_t b_t - c_t - T_t - a_t n)dt + a_t x_t dr_t^x$$

$$a_t = s_t + b_t + x_t a_t$$

- A representative household with members of initial size N_0 with $dN_t = ndt$ for $t > 0$
- s_t are safe assets with no liquidity yield, while b_t are government bonds with a liquidity yield

Fiscal Authority and Debt Dynamics

Government budget constraint and primary deficit:

$$dB_t = (r_t B_t + D_t)\, dt + \sigma_B B_t dZ_t^B$$
$$\frac{D_t}{N_t Y_t} = \frac{B_t}{N_t Y_t}\left[\alpha_d - \beta_d \log\left(\frac{B_t}{N_t Y_t}\right)\right]$$

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Lemma 1

The log debt to GDP ratio evolves as follows:

$$d\widehat{B}_t = \left(r_t - g - n + \alpha_d + \frac{\sigma_y^2 - \sigma_B^2}{2} - \beta_d \widehat{B}_t\right) dt + \sigma_{\widehat{B}} dZ_t^{\widehat{B}}$$
$$dZ_t^{\widehat{B}} = (\sigma_B / \sigma_{\widehat{B}}) dZ_t^B - (\sigma_y / \sigma_{\widehat{B}}) dZ_t^y$$
$$\sigma_{\widehat{B}}^2 = \sigma_B^2 + \sigma_y^2$$

Rates and Equity Premium

Interest rates and equity premium:

Rates and Equity Premium

Interest rates and equity premium:

$$r^s = \rho + \theta g - \frac{\gamma(\theta+1)}{2}\sigma_y^2$$

$$r_t = r^s - \alpha_u + \beta_u + \beta_u \widehat{B}_t$$

$$\frac{1}{dt}\mathbb{E}\left(dr_t^x - r^s\right) = \gamma\sigma_y^2$$

Rates and Equity Premium

Interest rates and equity premium:

$$
\begin{aligned}
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\end{aligned}
$$

$$
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Drift of the log debt to GDP ratio:

Rates and Equity Premium

Interest rates and equity premium:

$$\begin{aligned} r^s &= \rho + \theta g - \frac{\gamma(\theta+1)}{2}\sigma_y^2 \\ r_t &= r^s - \alpha_u + \beta_u + \beta_u \widehat{B}_t \end{aligned}$$

$$\frac{1}{dt}\mathbb{E}\left(dr_t^x - r^s\right) = \gamma\sigma_y^2$$

Drift of the log debt to GDP ratio:

$$\underbrace{\rho + (\theta-1)g - n}_{\text{deterministic}} - \underbrace{\frac{\gamma(\theta+1)}{2}\sigma_y^2}_{\text{risk}} + \underbrace{\frac{\sigma_y^2 - \sigma_B^2}{2}}_{\text{Ito's lemma}} - \underbrace{(\alpha_u - \beta_u)}_{\text{liquidity}} + \alpha_d - (\beta_d - \beta_u)\widehat{B}_t$$

Equilibrium Debt to GDP process

Proposition 2

If $\beta > 0$, the log debt to GDP ratio $\widehat{B}_t$ follows an Ornstein-Uhlenbeck process with:

$$d\widehat{B}_t = \left(\alpha - \beta\widehat{B}_t\right) dt + \sigma_{\widehat{B}} dZ_t^{\widehat{B}}$$

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Proposition 3

If $\beta > 0$, the log debt to GDP ratio admits a stationary distribution that is normal with:

$$\widehat{B} \sim \mathcal{N}\left(\frac{\alpha}{\beta}, \frac{\sigma_{\widehat{B}}^2}{2\beta}\right)$$

In levels, the debt to GDP ratio is lognormally distributed.

Comparative Statics

Mean and variance of the debt to GDP ratio:

$$\mathbb{E}\left(\frac{B_t}{N_t Y_t}\right) = e^{(\alpha+\sigma^2_{\widehat{B}})/\beta}$$
$$\mathbb{V}\left(\frac{B_t}{N_t Y_t}\right) = \left(e^{\sigma^2_{\widehat{B}}/2\beta} - 1\right) e^{2\alpha/\beta+\sigma^2_{\widehat{B}}/2\beta}$$

- Lower population growth n raises mean and variance of debt to GDP ratio
- Lower productivity growth g *lowers* mean and variance of the debt to GDP ratio when $\theta > 1$
- Effect of a rise in σ_y on mean debt to GDP ratio is ambiguous

Lifecycle model

Defining Debt Sustainability

- Assumed fiscal policy ensures existence of a stationary distribution for the debt to GDP ratio irrespective of drift term
- How should we think about debt dynamics absent an active fiscal response
- Allow the debt to GDP ratio to drift with a constant primary deficit
- Experiment in the spirit of Ball, Elmendorf and Mankiw (1998) and Blanchard (2019)

Distribution with Passive Fiscal Response

Proposition 4

If $\beta_d = \beta_u = 0$, $\alpha < 0$, and there exists a lower reflecting barrier, the process for the log debt to GDP ratio admits a stationary distribution that is an exponential distribution with rate parameter λ where:

$$d\widehat{B}_t = \alpha dt + \sigma_{\widehat{B}} dZ_t^{\widehat{B}}$$
$$\kappa = -2\alpha / \sigma_{\widehat{B}}^2$$

In levels, the stationary distribution of the debt-to-GDP ratio is Pareto with shape parameter κ.

Hitting a Default Threshold

- Both lognormal and Pareto distribution have an infinite support: $\mathbb{P}\left(b_t > b_{def}\right) > 0$

- Under passive fiscal response and given an initial debt to GDP ratio b_0, debt to GDP ratio will exceed $b_{def} > b_0$:

$$\lim_{t\to\infty} \mathbb{P}\left(b_t > b_{def}\right) = 1$$

- However, since log debt to GDP ratio is an ordinary Brownian motion under a passive fiscal response, expected first-passage time for *any* $b_{def} > b_0$ is infinite:

$$\mathbb{E}\left(T_{b_{def}}\right) = \infty$$

EXTENSIONS: RARE DISASTERS

Endowment process:

$$dY_t = gY_{t-} + \sigma_y Y_{t-} dZ_t^y + kY_{t-} dJ_t$$

Output follows a jump-diffusion process where $k < 0$ is the size of the fall in log output

Extensions: Rare Disasters

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Output follows a jump-diffusion process where $k < 0$ is the size of the fall in log output

Interest rates and equity premium (Wachter (2013)):

$$r^s = \rho + \theta g - \frac{\gamma(\theta+1)}{2}\sigma_y^2 + \lambda e^{-\gamma Z}\left(e^Z - 1\right)$$
$$\frac{1}{dt}\mathbb{E}\left(dr_t^x - r^s\right) = \gamma\sigma_y^2 + \lambda\left(e^{\gamma Z} - 1\right)\left(1 - e^Z\right)$$

where $k = e^Z - 1$ and λ is the intensity of the Poisson process J_t

Extensions: Rare Disasters

Stationary Distribution

Komolgorov forward equation:

$$0 = -\frac{d}{db}\alpha g(b) + \frac{1}{2}\frac{d^2}{db^2}\sigma_{\hat{b}}^2 g(b) - \lambda g(b) + \lambda g\left(be^{-Z}\right)$$
$$\Rightarrow 0 = \alpha\kappa + \frac{\sigma_{\hat{B}}^2}{2}\kappa(\kappa-1) - \lambda + \lambda e^{Z(\kappa+1)}$$

Proposition 5

With rare disasters, the debt to GDP ratio follows a geometric Brownian motion with jumps. If there exists a $\kappa > 0$ that solves the KFE, the debt to GDP ratio admits a stationary distribution that is Pareto with tail parameter κ.

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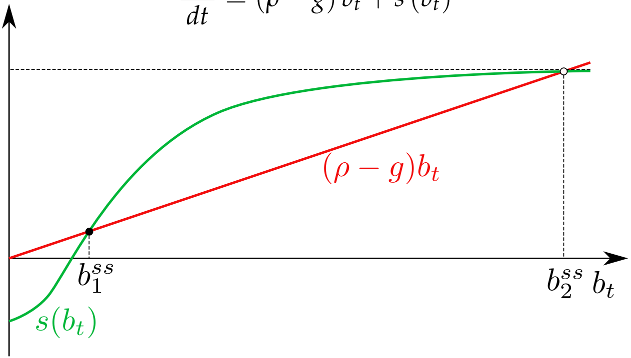
Debt Dynamics under Default

Deterministic Case

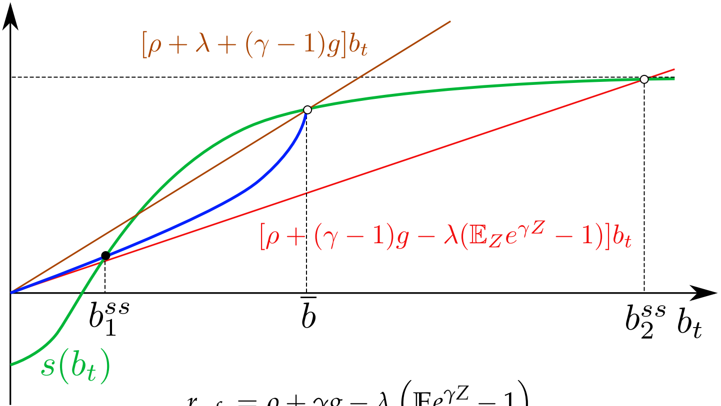
$$\frac{db_t}{dt} = (\rho - g)\, b_t + s\,(b_t)$$



- Surplus function $s\,(\cdot)$ is bounded above
- Maximum surplus motivated by presence of a Laffer curve

Debt Dynamics under Default

Rare disasters

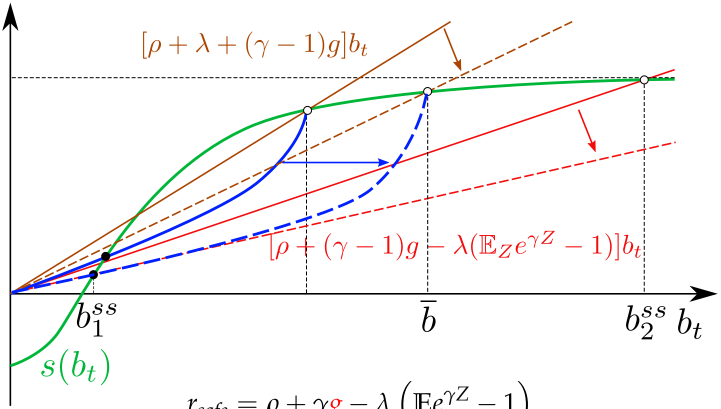


$$r_{safe} = \rho + \gamma g - \lambda \left(\mathbb{E} e^{\gamma Z} - 1 \right)$$

$$r_t = r_{safe} + \underbrace{\lambda p \left(b_t, \bar{b} \right) \mathbb{E} e^{\gamma Z}}_{\text{default premia}}$$

Debt Dynamics under Default

Decline in growth

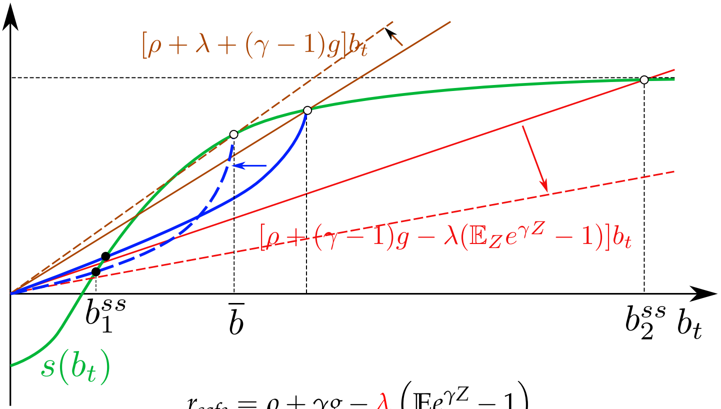


$$r_{safe} = \rho + \gamma g - \lambda \left(\mathbb{E}e^{\gamma Z} - 1\right)$$

$$r_t = r_{safe} + \lambda p\left(b_t, \bar{b}\right)\mathbb{E}e^{\gamma Z}$$

Debt Dynamics under Default

Rise in disaster risk



$$r_{safe} = \rho + \gamma g - \lambda \left(\mathbb{E} e^{\gamma Z} - 1\right)$$

$$r_t = r_{safe} + \lambda p \left(b_t, \bar{b}\right) \mathbb{E} e^{\gamma Z}$$

Key Takeaways

Lessons:

- Average cost of servicing the debt close to zero or negative
- Elevated risk of rare disasters may be *beneficial* for debt sustainability by lowering servicing cost
- With default, elevated risk premia lowers debt limit but also lowers safe interest rate

Limitations:

- $r - (g + n)$ not a sufficient statistic for optimal level of debt
- Optimal level of debt depends on degree of crowding out, costs of distortionary taxation, etc.

Additional Slides

Calibration Strategy

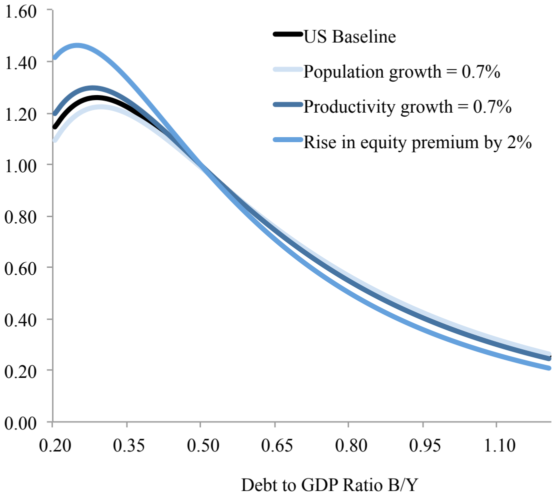
1. Output process: $g = 2.06\%$, $\sigma_y = 2.5\%$, $n = 1.15\%$
2. Elasticity of intertemporal substitution: $1/\theta = 0.75$
3. Liquidity parameters: α_u, β_u
   - Regression of spread on US AAA corporate debt relative to 10-year Treasuries on debt to GDP ratio (Krishnamurthy and Vissing-Jorgensen (2012))
4. Safe rate and equity premium: ρ and γ
   - Target gov't bond yield of 2.48% and equity premium of 5.16% (Jorda et al. (2018))
5. Fiscal policy parameters: $\alpha_d, \beta_d, \sigma_b$
   - Target mean and variance of log debt to GDP ratio in postwar period (Jorda, Schularick and Taylor (2016))
   - Target correlation of r_t and dY_t/Y_t of -0.056 in postwar period

Secular Stagnation Effects

Comparative Statics

Panel A: Active fiscal response	$\mathbb{E}r_t$	$\frac{1}{dt}\mathbb{E}\left(dr_t^x - r_t\right)$	$\mathbb{E}b_t$	$\mathbb{V}b_t$
Baseline	2.48	5.16	-	-
Pop. growth $n = 0.70\%$	2.48	5.16	+16%	+33%
Prod. growth $g = 0.70\%$	0.80	5.16	-13%	-25%
Rise in risk premia σ_y	0.16	7.16	-33%	-29%
Panel B: Passive fiscal response	α	λ		
Baseline	-0.73%	1.071		
Pop. growth $n = 0.70\%$	-0.28%	1.027		
Prod. growth $g = 0.70\%$	-1.18%	1.117		
Rise in risk premia	-3.03%	1.115		

Shifts in Stationary Distribution



Rare Disasters

Comparative Statics

- Calibrate rare disaster probability: $\delta = 1.7\%$ and loss $k = -29\%$ based on Barro (2006)
- Resulting risk aversion coefficient: $\gamma = 7$

Passive fiscal response	$\mathbb{E}r_t$	$\frac{1}{dt}\mathbb{E}\left(dr_t^x - r_t\right)$	λ
Baseline	2.48	5.36	0.968
Pop. growth $n = 0.70\%$	2.48	5.36	0.921
Prod. growth $g = 0.70\%$	0.67	5.36	1.015
Rise in risk premia $\delta = 2.4\%$	0.25	7.39	1.161
Rise in risk premia $k = -31.4\%$	0.43	7.37	1.172

Economic Environment

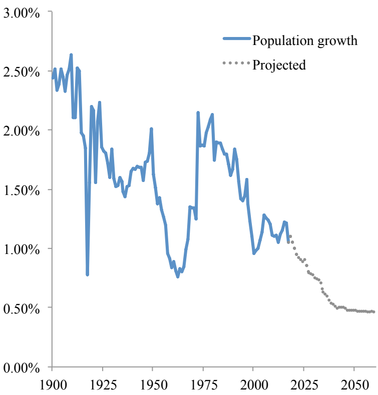
- Time: $t = 0, 1, 2, ...$
- Goods: consumption and investment good
- Agents: households (J cohorts), representative firm
- Assets: capital, bonds
- Technology: age-specific human capital profiles hc_j

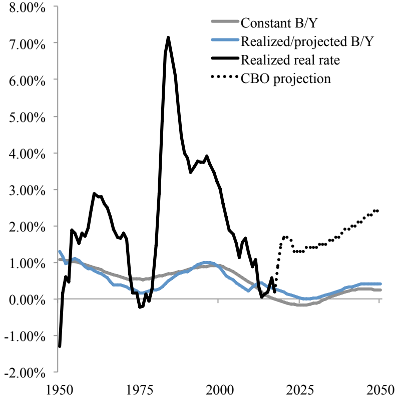
Calibration and Targeted Moments

Panel A: Data	*Symbol*	*Value*	*Source*
Mortality profile	$s_{j,t}$		US mortality tables, CDC
Income profile	hc_j		Gourinchas and Parker (2002)
Population growth rate	n	0.70%	US Census Bureau
Productivity growth	g	0.70%	Fernald (2012)
Government spending (% of GDP)	$\frac{G}{Y}$	19.2%	BEA
Public debt (% of GDP)	$\frac{b_g}{Y}$	70%	CBO
Panel B: Related literature	*Symbol*	*Value*	
Elasticity of intertemporal substitution	ρ	0.75	
Depreciation rate	δ	8%	
Panel C: Matching targets	*Symbol*	*Value*	*Target*
Rate of time preference	β	1.0029	Real US 10-year rate
Intermediation wedge	ω	0.1733	Corporate Aaa spread
Retailer elasticity of substitution	θ	4.6174	Labor share
Capital share parameter	α	0.2341	Investment to GDP ratio

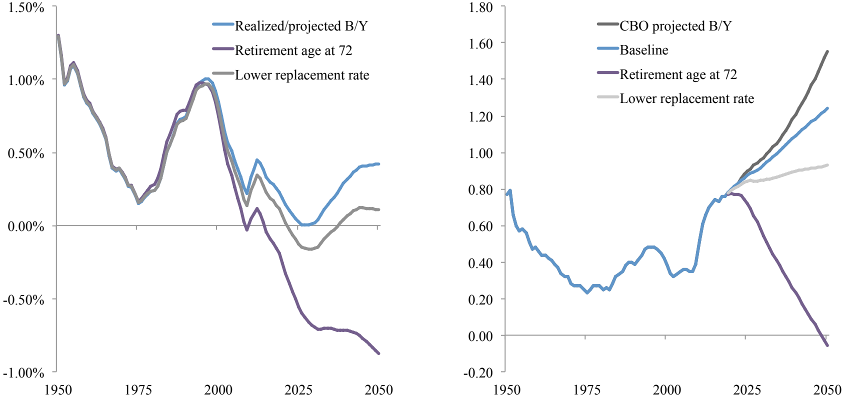
- Social security replacement rate of 50% and retirement at age 65
- Age and survival probabilities based on Census projections

EFFECT OF AGING ON INTEREST RATES





Debt to GDP projections for the US



- Baseline model projection more optimistic than CBO
- Social security reforms have large impacts on the debt to GDP ratio

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