Hands-Off Control as Green Control

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***Abstract*—In this article, we introduce a new paradigm of control, called hands-off control, which can save energy and reduce CO2 emissions in control systems. A hands-off control is defined as a control that has a much shorter support than the horizon length. The maximum hands-off control is the minimum support (or sparsest) control among all admissible controls. With maximum hands-off control, actuators in the feedback control system can be stopped during time intervals over which the control values are zero. We show the maximum hands-off control is given by L^1 optimal control, for which we also show numerical computation formulas.**

I. Introduction

In practical control systems, we often need to minimize the control effort so as to achieve control objectives under limitations in equipment such as actuators, sensors, and networks. For example, the energy (or L^2-norm) of a control signal is minimized to prevent engine overheating or to reduce transmission cost with a standard LQ (linear quadratic) control problem; see e.g., [1]. Another example is the *minimum fuel* control, discussed in e.g., [3], in which the total expenditure of fuel is minimized with the L^1 norm of the control.

Alternatively, in some situations, the control effort can be dramatically reduced by holding the control value *exactly zero* over a time interval. We call such control a *hands-off control*. A motivation for hands-off control is a stop-start system in automobiles. It is a hands-off control; it automatically shuts down the engine to avoid it idling for long periods of time. By this, we can reduce CO or CO2 emissions as well as fuel consumption [7]. This strategy is also used in hybrid vehicles [5]; the internal combustion engine is stopped when the vehicle is at a stop or the speed is lower than a preset threshold, and the electric motor is alternatively used. Thus hands-off control is also available for solving environmental problems. Hands-off control is also desirable for networked and embedded systems since the communication channel is not used during a period of zero-valued control. This property is advantageous in particular for wireless communications [9]. In other words, hands-off control is the least *attention* in such periods. From this point of view, hands-off control that maximizes the total time of no attention is somewhat related to the concept of minimum attention control [4].

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Motivated by these applications, we propose a new paradigm of control, called *maximum hands-off control* that maximizes the time interval over which the control is exactly zero. Although this type of optimization is highly non-convex, we have proved in [11] that under the normality assumption on the optimal control problem, the maximum hands-off control is given by L^1 optimal control, which can be solved much more easily [3].

II. Optimal Control Problems

We here consider nonlinear plant models of the form

$$\frac{d\boldsymbol{x}(t)}{dt} = \boldsymbol{f}\big(\boldsymbol{x}(t)\big) + \sum_{i=1}^{m} \boldsymbol{g}_i\big(\boldsymbol{x}(t)\big)u_i(t), \quad t \in [0,T], \tag{1}$$

where $\boldsymbol{x}$ is the state, $u_1,\dots,u_m$ are the control inputs, $\boldsymbol{f}$ and $\boldsymbol{g}_i$ are functions on $\mathbb{R}^n$. We assume that $\boldsymbol{f}(\boldsymbol{x})$, $\boldsymbol{g}_i(\boldsymbol{x})$, and their Jacobians $\boldsymbol{f}'(\boldsymbol{x})$, $\boldsymbol{g}_i'(\boldsymbol{x})$ are continuous in $\boldsymbol{x}$. We use the vector representation $\boldsymbol{u} \triangleq [u_1,\dots,u_m]^\top$.

The control $\{\boldsymbol{u}(t): t\in[0,T]\}$ is chosen to drive the state $\boldsymbol{x}(t)$ from a given initial state

$$\boldsymbol{x}(0) = \boldsymbol{x}_0, \tag{2}$$

to the origin by a fixed final time $T>0$, that is,

$$\boldsymbol{x}(T) = \boldsymbol{0}. \tag{3}$$

Also, the control $\boldsymbol{u}(t)$ is constrained in magnitude by

$$\|\boldsymbol{u}(t)\|_\infty \leq 1, \quad \forall t \in [0,T]. \tag{4}$$

We call a control $\{\boldsymbol{u}(t): t\in[0,T]\}$ *admissible* if it satisfies (4) and the resultant state $\boldsymbol{x}(t)$ from (1) satisfies boundary conditions (2) and (3). We denote by $\mathcal{U}$ the set of all admissible controls.

The *maximum hands-off control* is a control that maximizes the time interval over which the control $\boldsymbol{u}(t)$ is exactly zero. In other words, we try to find the *sparsest* control among all admissible controls in $\mathcal{U}$.

We state the associated optimal control problem as follows:

Problem 1 (Maximum Hands-Off Control): Find an admissible control $\{\boldsymbol{u}(t): t\in[0,T]\}\in\mathcal{U}$ that minimizes

$$J_0(\boldsymbol{u}) \triangleq \sum_{i=1}^{m} \lambda_i \|u_i\|_{L^0}, \tag{5}$$

where $\lambda_1>0,\dots,\lambda_m>0$ are given weights.

On the other hand, if we replace $\|u_i\|_{L^0}$ in (5) with the L^1 norm $\|u_i\|_{L^1}$, we obtain the following L^1-*optimal control* problem, also known as *minimum fuel control* discussed in e.g. [2], [3].

Problem 2 (L^1-Optimal Control): Find an admissible control $\{\boldsymbol{u}(t) : t \in [0,T]\} \in \mathcal{U}$ that minimizes

$$J_1(\boldsymbol{u}) \triangleq \sum_{i=1}^{m} \lambda_i \|u_i\|_{L^1} = \int_0^T \sum_{i=1}^{m} \lambda_i |u_i(t)| dt, \qquad (6)$$

where $\lambda_1 > 0, \ldots, \lambda_m > 0$ are given weights.

III. Maximum Hands-Off Control and L^1-Optimal Control

In this section, we consider a theoretical relation between maximum hands-off control (Problem 1) and L^1-optimal control (Problem 2). The theorem below rationalizes the L^1 optimality in computing the maximum hands-off control [11].

Theorem 3: Assume that the L^1-optimal control problem stated in Problem 2 is normal[1] and has at least one solution. Let $\mathcal{U}_0^*$ and $\mathcal{U}_1^*$ be the sets of the optimal solutions of Problem 1 (L^0-optimal control problem) and Problem 2 (L^1-optimal control problem) respectively. Then we have $\mathcal{U}_0^* = \mathcal{U}_1^*$.

Theorem 3 suggests that L^1 optimization can be used for the maximum hands-off (or the sparsest) solution. This is analogous to the situation in compressed sensing, where L^1 optimality is often used to obtain the sparsest vector; see [8] for details.

IV. Linear Plants and Numerical Computation

We here propose a numerical computation method to obtain an L^1-optimal control (i.e. maximum hands-off control) when the plant model is linear and time-invariant.

Let us consider the following linear time-invariant plant model

$$\frac{d\boldsymbol{x}(t)}{dt} = A\boldsymbol{x}(t) + B\boldsymbol{u}(t), \quad t \in [0,T], \quad \boldsymbol{x}(0) = \boldsymbol{x}_0, \qquad (7)$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ and $\boldsymbol{u}(t) \in \mathbb{R}^m$. We assume that the initial state $\boldsymbol{x}_0 \in \mathbb{R}^n$ and the time $T > 0$ are given.

Linear systems are much easier to treat than general nonlinear systems as in (1). In particular, for special plants, such as single or double integrators, the L^1-optimal control can be obtained analytically; see e.g., [3, Chap. 8]. However, for general linear time-invariant plants, one should rely on numerical computation. For this, we adopt a time discretization approach to solve the L^1-optimal control problem. This approach is standard for numerical optimization; see e.g. [13, Sec. 2.3].

We first divide the interval $[0,T]$ into N subintervals, $[0,T] = [0,h) \cup \cdots \cup [(N-1)h, Nh]$, where h is the discretization step chosen such that $T = Nh$. We here assume (or approximate) that the state $\boldsymbol{x}(t)$ and the control $\boldsymbol{u}(t)$ are constant over each subinterval. On the discretization grid, $t = 0, h, \ldots, Nh$, the continuous-time plant (7) is described as

$$\boldsymbol{x}_d[m+1] = A_d \boldsymbol{x}_d[m] + B_d \boldsymbol{u}_d[m], \quad m = 0, 1, \ldots, N-1,$$

where $\boldsymbol{x}_d[m] \triangleq \boldsymbol{x}(mh)$, $\boldsymbol{u}_d[m] \triangleq \boldsymbol{u}(mh)$, and

$$A_d \triangleq e^{Ah}, \quad B_d \triangleq \int_0^h e^{At} B dt.$$

[1]When the optimal control is uniquely determined almost everywhere from the minimum principle, the control problem is called normal. See [3] for details.

Set the control vector

$$\boldsymbol{U} \triangleq [\boldsymbol{u}_d[0]^\top, \boldsymbol{u}_d[1]^\top, \ldots, \boldsymbol{u}_d[N-1]^\top]^\top.$$

Note that the final state $\boldsymbol{x}(T)$ can be described as

$$\boldsymbol{x}(T) = \boldsymbol{x}_d[N] = A_d^N \boldsymbol{x}_0 + \Phi_N \boldsymbol{U},$$

where

$$\Phi_N \triangleq \left[A_d^{N-1} B_d, \quad A_d^{N-2} B_d, \quad \ldots, \quad B_d \right].$$

If we define the following matrices:

$$\Lambda_m \triangleq \mathrm{diag}(\lambda_1, \ldots, \lambda_m), \ \Lambda \triangleq \mathrm{blockdiag}(\underbrace{\Lambda_m, \ldots, \Lambda_m}_{N}),$$

then the L^1-optimal control problem is approximately described as

$$\begin{aligned} &\underset{\boldsymbol{U} \in \mathbb{R}^{mN}}{\text{minimize}} && \|\Lambda \boldsymbol{U}\|_1 \\ &\text{subject to} && \|\boldsymbol{U}\|_\infty \leq 1, \ A_d^N \boldsymbol{x}_0 + \Phi_N \boldsymbol{U} = \boldsymbol{0}. \end{aligned} \qquad (8)$$

The optimization problem (8) is convex and can be efficiently solved by numerical software packages such as `cvx` with MATLAB; see [6] for details.

V. Conclusion

In this article, we have presented maximum hands-off control and shown that it is L^1 optimal. This shows that efficient optimization methods for L^1 problems can be used to obtain maximum hands-off control. A time discretization method has been presented for the computation of L^1-optimal control when the plant is linear time-invariant. The resultant optimization is a convex one, and hence can efficiently be solved. Future work may include adaptation of hands-off control to sparsely packetized predictive control as in [10], [12].

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